Tracer vs. Pressure Wave Velocities through Unsaturated Saprolite

Todd C. Rasmussen,* Roger H. Baldwin, Jr., John F. Dowd, and Andrew G. Williams

ABSTRACT

Saprolite is a form of weathered bedrock that is commonly used as the host material at waste disposal sites in the Southeastern Piedmont. However, estimating the unsaturated hydraulic and transport properties of saprolite is difficult due to saprolite’s low permeability. We demonstrate the use of short-duration fluid irrigation pulses for maintaining unsaturated conditions in intact saprolite columns. Complementary tracer experiments demonstrate that irrigated waters moved through an effective volumetric porosity (0.038±0.108 cm$^3$ cm$^{-3}$) substantially less than the ambient water-filled porosity (0.44 cm$^3$ cm$^{-3}$). We observed the unexpected result that irrigation-induced pressure wave velocities (1983±3670 cm d$^{-1}$) were ~1000 times faster than tracer velocities (2.04±6.00 cm d$^{-1}$). The relationship between pressure wave velocities and fluid velocities is described using kinematic wave theory, presented for four parametric representations (Brooks–Corey, van Genuchten–Mualem, Broadbridge–White, and the Galileo Number), that predicts fluid pressure velocities to be from approximately two to fifteen times faster than saprolite tracer velocities. None of the kinematic models was able to reproduce observed rapid pressure wave velocities. A hydraulic form of the advection–diffusion equation based on Richards’ equation is presented that favorably predicts the shape of pressure response curves only when the kinematic velocity is ignored and the hydraulic diffusivity of the unsaturated saprolite is considered. Based on the advection–diffusion equation, diffusion-dominated soil water pressure wave velocities should decrease with depth, eventually conforming with kinematic wave theory. Pressure pulse velocity monitoring may be an additional tool for estimating unsaturated hydraulic properties in low permeability media.

Water potential and tracer travel times through unsaturated media are often faster than predictions (Steenhuis et al., 1994). Explanations for rapid tracer movement are ascribed to preferential flow (Germann, 1990; Mohanty et al., 1998), bypass flow (Booltink and Bouma, 1993; Booltink et al., 1993; Booltink, 1994), macropore flow (Beven and Germann, 1982; Bouma, 1982; Bouma et al., 1982; Germann and Beven, 1985; Lichner, 1997), and fracture flow (Gimm et al., 1997). Additional explanations include boundary layer flow (Germann, 1990; Alauoi et al., 1997), mobile zone flow (Clothier et al., 1995; Casey et al., 1998; Vogeler et al., 1998), finger flow (Liu et al., 1994, 1995), funnel flow (Kung, 1993; Ju and Kung, 1997), media heterogeneities (Yeh et al., 1985), ion exclusion (Sato et al., 1992), and colloid transport (Corapcioglu and Choi, 1996). These explanations posit that rapid transport primarily occurs due to preferential movement through a subdomain embedded within the global flow domain. Observed tracer breakthrough curves are often used to estimate model parameters, including effective porosity, finger width, mobile zone porosity, and the retardation factor. Each approach provides valuable information about the behavior of solute transport through unsaturated media.

Water potential responses are also used to estimate hydraulic parameters (Inoue et al., 1998). In these cases, perturbations at the surface, from precipitation and irrigation, and within the subsurface, from fluid injections or removals, are monitored and used for parameter estimation. The unsaturated hydraulic conductivity, hydraulic diffusivity, or sorptivity are determined in these cases. Perturbations due to fluid injection or extraction, or the modification of the pressure head at a surface or point, induce a step or spike change in fluid pressure that is transmitted through the unsaturated zone. The pressure wave response, also called the kinematic model, has been used to model subsurface stormflow (Beven, 1982), vertical flow through unsaturated soils (Philip, 1957; Sisson et al., 1980; Smith, 1983; Charbe-nea, 1984; Germann and Beven, 1985; Germann, 1990), overland flow (Kibler and Woolhiser, 1972; Cundy and Tento, 1985), and percolation through snow (Colbeck, 1972). Many other studies (Wierenga, 1977; Koide and Wheeler, 1992) have been conducted on the effects of transient surface irrigation and precipitation responses. These analyses focused on predicting water content changes and water movement in response to dynamic changes in boundary flux.

Saprolite results from in situ weathering of parent material, forming the dominant C horizon in the Southeastern Piedmont. Previous studies of fluid flow and transport through saprolite (Vepraskas and Williams, 1995; Williams et al., 1994) were conducted because of waste disposal concerns. Schoeneberger and Amooze-gar (1990) examined saprolite in a Cecil (fine, kaolinitic, thermic Typic Kanhapludult) soil near Raleigh, North Carolina, and showed that fractured, planar, quartz veins appeared to be the most active component of macropore flow. However, foliation planes within the saprolite did not significantly affect either the saturated hydraulic conductivity or macropore flow at the site. Vepraskas et al. (1991) examined a mica-schist saprolite from the Schenck Forest near Raleigh, NC. Undisturbed cores were used to show that water flow along foliation planes was negligible, but that old root channels contributed significantly to flow near saturation.

Cook et al. (1996) examined flow and tracer behavior in fractured saprolite at Oak Ridge National Laboratory in Tennessee and found rapid equilibration between fractures and weathered bedrock. Rapid, fracture-dominated flow was observed in unweathered rock. Reedy et al. (1996) investigated flow and transport through fractured, shale saprolite in the Melton Branch Watershed at the Oak Ridge Reservation in Tennessee. Bro-
mide tracer migration within an undisturbed saprolite column was affected by the magnitude of and duration between episodic flow interruptions. Increased diffusion between fracture macropores and the soil matrix was used to account for reduced vertical Br\textsuperscript{-} migration through the soil column.

The primary objective of this study was to develop a procedure for obtaining hydraulic and tracer properties of intact columns of unsaturated saprolite. Efforts to reduce steady flow rates sufficient to maintain unsaturated conditions proved to be unsuccessful, however, because of the low unsaturated hydraulic conductivity of saprolite. Instead, we found that short-duration irrigations at the upper surface of saprolite columns provided water pressure heads that were consistent with unsaturated conditions. A secondary objective was to explain the extremely rapid pressure wave velocities obtained using short-duration irrigations. Wave velocity predictions using darcian, tracer, and kinematic models substantially underestimated observed travel times.

**Approach**

We focus our examination of fluid flow and transport behavior on the effects of small flow perturbations at the surface. Fluid flow through unsaturated media is characterized using the vector form of Richards’ equation:

\[
\nabla q = \nabla (-K \nabla H) = \frac{\partial \theta}{\partial t}
\]

where \(q\) is the darcian flux vector, \(K\) is the water-dependent hydraulic conductivity, \(\nabla H\) is the total head gradient, \(\theta\) is the volumetric soil water content, and \(t\) is time. Eq. [1] in one-dimensional form is:

\[
\frac{\partial}{\partial z} \left( K \frac{\partial H}{\partial z} \right) = \frac{\partial \theta}{\partial t}
\]

The fluid or tracer velocity, \(v\), is related to the darcian flux using:

\[
v = \frac{q}{\theta} = -\frac{K}{\theta} \nabla H
\]

The fluid velocity can be experimentally determined in the field and laboratory using \(v = \Delta t/\tau\), where \(\Delta t\) is the travel distance and \(\tau\) is the median travel time, obtained by noting the arrival time of one-half of the mass of a conservative tracer. Observed tracer velocities may be inconsistent with fluid velocities computed using Eq. [3] because of many physical and chemical processes that affect tracer mobility, including absorption–desorption, diffusion, hydrodynamic dispersion, and others. Another reason for this discrepancy, demonstrated next, occurs when the darcian flux is confused with the pressure wave velocity.

The kinematic velocity, \(c\), (also called the celerity or wave velocity) is defined as the derivative of the darcian flux with respect to the water content (Sisson et al., 1980; Smith, 1983; Singh, 1997):

\[
c = \frac{dq}{d\theta}
\]

This formulation can be obtained by rearranging Richards’ equation (Eq. [1]):

\[
\frac{dq}{d\theta} = \frac{dz}{dt}
\]

and substituting Eq. [4] to obtain:

\[
c = \frac{dz}{dr}
\]

which is the Lagrangian velocity of the fluid pulse. The kinematic velocity is used to predict the rate of movement of a pressure or energy perturbation through the medium. Unlike saturated media where pressure waves propagate due to the compressibility of the fluid, pressure waves propagate through unsaturated media as a result of small changes in fluid saturation within soil pores.

If the unsaturated zone pressure perturbation is small and does not substantially affect the vertical, gravity-driven, hydraulic gradient, we have:

\[
c = \frac{dK}{d\theta} \nabla H
\]

For conditions of steady, vertical, gravity-driven flow through the unsaturated zone (where \(z\) is taken in the downward direction for convenience) a unit hydraulic gradient, \(\nabla H = [0,0,-1]\), is conventionally specified (Sisson et al., 1980), so that:

\[
q = K
\]

\[
v = \frac{K}{\theta}
\]

\[
c = \frac{dK}{d\theta}
\]

Equation [10] is consistent with the limit of the Philip wetting front velocity, \(c = \Delta K/\Delta \theta\), for small changes in water content, where \(\Delta K\) and \(\Delta \theta\) are, respectively, the changes in unsaturated hydraulic conductivity and water content across the wetting front (Philip, 1957; Jury et al., 1991).

The kinematic ratio, \(k = c/v\), is used to describe the velocity of the kinematic velocity relative to the fluid velocity, which, when combined with Eq. [9] and [10], yields:

\[
k = \frac{c}{v} = \frac{\theta}{K} \frac{dK}{d\theta} = \frac{d \ln K}{d \ln \theta}
\]

which implies that the kinematic ratio is the slope of the log-log plot between \(K\) and \(\theta\). Values of \(k\) less than unity indicate that the velocity of a small pressure perturbation is slower than the fluid velocity, while values of \(k\) greater than unity indicate that the pressure wave is faster than the fluid velocity.

Equation [7] assumes that the hydraulic gradient is unaffected by the pressure wave. A more robust analysis of the movement of a kinematic wave through a one-
dimensional vertical column can be obtained by manipulation of Richards' equation (Eq. [1]):

$$\frac{\partial}{\partial z} \left( K \frac{\partial H}{\partial z} \right) = C_p \frac{\partial \psi}{\partial t} \quad [12]$$

where $H = -z + \psi$ is the total head, $\psi$ is the soil water pressure head, $C_p = d\psi/d\psi$ is the soil water capacity, and $dH/\partial t = d\psi/\partial t$ because $\partial^2 z/\partial t = 0$. Application of the chain rule yields:

$$K \frac{\partial^2 H}{\partial z^2} + \frac{\partial K}{\partial z} \frac{\partial H}{\partial z} = C_p \frac{\partial \psi}{\partial t} \quad [13]$$

or

$$D \frac{\partial^2 \psi}{\partial z^2} - \frac{\partial \psi}{\partial z} \frac{\partial \psi}{\partial t} = \frac{\partial \psi}{\partial t} \quad [14]$$

because:

$$\frac{\partial K}{\partial z} = \frac{dK}{d\theta} d\psi / dz = c C_p \frac{d\psi}{dz} \quad [15]$$

where $D = K/C_p$ is the hydraulic diffusivity and $c = dK/d\theta$ is the conventional definition of wave velocity; $\partial H/\partial z = -1$ is the hydraulic gradient, assuming that $\partial \psi/\partial z >> \partial \psi/\partial z$, and $\partial^2 H/\partial z^2 = \partial^2 \psi/\partial z^2$. Equation [14] is mathematically identical to the advection–diffusion equation used in solute transport. In this case, the soil water pressure head replaces the solute concentration, the hydraulic diffusion coefficient replaces the solute diffusion, or dispersion, coefficient, and the kinematic velocity replaces the tracer velocity.

The solution of the advection–diffusion equation for a dirac (or spike) input, $\psi(0,t) = C_s \delta(t)$, along a semi-infinite boundary is found in Leij and Toride (1995):

$$\psi(z,t) = \frac{z C_s}{\sqrt{4\pi D t^3}} \exp \left[ - \frac{(z - ct)^2}{4Dt} \right] \quad [16]$$

where $\psi(z,t)$ is the depth- and time-dependent soil water pressure head response, $C_s$ is the hydraulic condition at the upper boundary, and $\delta(t)$ is the dirac function. This formulation requires constant hydraulic diffusivity and kinematic velocity, which is consistent with small perturbations to the water content. The time of peak response, $t_p$, occurs when the time derivative of the soil water pressure head equals zero; that is, $d\psi/\partial t = 0$, which occurs for a spike input when:

$$z^2 - c^2 t_p^2 = 6D t_p \quad [17]$$

The observed velocity of the wave peak, $w$, is:

$$w = \frac{z}{t_p} = c \frac{\kappa}{\sqrt{9 + \kappa^2}} \quad [18]$$

where $\kappa = c z/D$ is a hydraulic form of the dimensionless Peclet number. The limits of the velocity of the observed pressure wave peak are:

$$w = \frac{6D}{z} \quad \kappa << 1 \quad [19a]$$

$$w = c \quad \kappa >> 1 \quad [19b]$$

The hydraulic Peclet number, $\kappa$, reduces to:

$$\kappa = \frac{cz}{D} = \frac{(dK/d\theta)z}{K/C_p} = \frac{d \ln K}{d\eta} z \quad [20]$$

because $C_p = d\psi/d\psi$. The hydraulic Peclet number increases linearly with depth, but is also a function of the slope of a semilogarithmic plot between the soil water pressure head and the unsaturated hydraulic conductivity.

Equation [18] implies rapid wave velocities close to the source due to the diffusion of the pressure wave, with asymptotic behavior at some distance beyond $z >> D/c$, due to advection of the pressure wave. Thus, pressure wave formulations that utilize Eq. [19b] alone may underestimate pressure velocities, especially near the source of the perturbations. Likewise, formulations that utilize Eq. [19a] alone may underestimate velocities at large distances below the perturbation.

### Parametric Forms of the Kinematic Equation

Parametric expressions for the kinematic velocity, $c$, and the kinematic ratio, $k$, can be obtained for simple expressions such as the Brooks–Corey (Brooks and Corey, 1964), van Genuchten–Mualem (van Genuchten, 1978, 1980), and Broadbridge–White (Broadbridge and White, 1988) formulations. The water retention, fluid flux, tracer velocity, kinematic velocity, and kinematic ratio for these formulations are provided in Table 1 for conditions where gravitational flow dominates and the residual saturation is zero.

The Brooks–Corey kinematic velocity is equivalent to that found in Sisson et al. (1980). The Brooks–Corey formulation implies that the kinematic velocity increases with increasing saturation, meaning that the wave velocity accelerates as the soil becomes wetter. It is important to note that the ratio of the kinematic velocity to the fluid velocity (i.e., the kinematic ratio) is independent of water content, meaning that the kinematic ratio changes at the same rate as the fluid velocity. For most soils, $k > 1$, indicating that a fluid pressure pulse moves through a soil faster than the average fluid velocity. The van Genuchten–Mualem formulation predicts that $k = 1/2 + 1/m$ for $\Theta = 0$, and $k = \infty$ for $\Theta = 1$, which indicates that kinematic velocities are maximal near saturation, and larger than the fluid velocity under dry conditions as long as $m \leq 2$. The Broadbridge–White formulation results in a kinematic ratio that equals two when $\Theta = 0$, and $k = 2 + 1/(C - 1)$ for $\Theta = 1$, which implies large kinematic ratios for saturated conditions when $C$ approaches unity.

Dimensional analysis is routinely used in fluid mechanics to understand and predict fluid flow behavior (Gerhart et al., 1992). A formulation is now presented that copies the approach used in surface water hydrology to predict flood wave velocities. The surface-water approach uses the dimensionless Froude number (the ratio between inertial and gravitational forces) to predict the kinematic ratio (Rouse, 1946). Within the unsaturated zone, it is clear that while gravitational forces are still present, inertial forces no longer dominate. We propose that the dimensionless Galileo number (the
ratio between gravitational and viscous forces) is a reasonable starting point for predicting the kinematic ratio.

The gravitational force is found by noting that the downward force on water exerted by gravity, $F_g$, can be calculated using:

$$F_g = \gamma V_v$$  \[21\]

where $\gamma$ is the fluid specific weight and $V_v$ is the fluid volume. We next calculate the viscous force resisting the downward movement, $F_v$, by applying Newton’s formula:

$$F_v = \tau A_p = \frac{\mu \nu A_p^2}{V_v}$$  \[22\]

where $\tau = \mu \nu / R$ is the shear stress applied across the wetted surface area, $A_p$, and where $\mu$ is the dynamic viscosity, $\nu$ is the fluid velocity defined earlier, and $R = V_v / A_p$ is the hydraulic radius.

As noted above, the ratio of the gravitational to viscous forces is the dimensionless Galileo number, $Ga$:

$$Ga = \frac{F_g}{F_v} = \frac{\gamma V_v}{\mu \nu A_p^2}$$  \[23\]

This expression can be simplified by dividing both the fluid volume and the particle surface area by the bulk volume, $V_f$, yielding:

$$Ga = \frac{\gamma}{\nu A_p^2}$$  \[24\]

where $A_p$ is the specific surface area.

We propose that the Galileo number be used to predict unsaturated pressure wave velocities. This approach implies that the gravity-driven downward velocity of a pressure wave is resisted by viscous forces, resulting in:

$$Ga = k = \frac{c}{\nu} = \frac{\gamma}{\nu A_p^2}$$  \[25\]

or

$$c = \frac{\gamma}{\nu A_p^2}$$  \[26\]

This approach is consistent with surface-water flood wave predictions that use the Froude number to predict the flood wave kinematic ratio. We test this approach by applying $c = dK/d\theta$ to determine the unsaturated hydraulic conductivity function:

$$K(\theta) = \int_0^\theta c \ d\theta = K_c \theta^b$$  \[27\]

where we assume that all coefficients are constant except for the water content, and where:

$$K_c = \frac{1}{3} \frac{\gamma}{\mu A_p^2}$$  \[28\]

which is consistent with the Kozeny–Carmen equation (Bear, 1988), but neglects interfacial tension effects at the air–water–solid interface (Lu et al., 1994).

We further test this approach by noting that Eq. [27] is equivalent to the Brooks–Corey formulation when $b = 3$, or $\lambda = \infty$. Unfortunately, the Brooks–Corey water retention function is undefined for this value of $\lambda$. Additional forces, such as surface tension acting on the meniscus, may also affect the pressure wave velocity.

The fluid flux, tracer velocity, kinematic velocity, and kinematic ratio as a function of relative saturation for the Galileo number formulation are presented in Table 1. It is interesting to note that the kinematic ratio calculated using the Galileo number, like the Brooks–Corey relationship, is constant for all water contents.

### MATERIALS AND METHODS

Three intact saprolite columns were extracted from the University of Georgia Research Farm near Watkinsville, GA, in the manner of Tindall et al. (1992). Saprolite columns were 30 cm in diameter and 38 cm in height, resulting in a surface area of 707 cm$^2$. The large column size was important because of substantial heterogeneity in the parent material. The saprolite is weathered from the Athens Gneiss, has a sandy-loam texture, no discernable structure or visible macropores, occasional quartz veins (ranging from 2 to 4 cm in width), and visible foliation derived from the parent rock. Color of the weathered gneiss is a deep red, while the foliation color ranges from white through yellow to black. Overlying soils belong to
Table 2. Experimental configuration for intact saprolite columns showing depths of sampling devices.

<table>
<thead>
<tr>
<th>Columns 1 and 2</th>
<th>Depth</th>
<th>Column 3</th>
<th>Depth</th>
</tr>
</thead>
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<tr>
<td>Saprolite surface</td>
<td>0</td>
<td>Saprolite surface</td>
<td>0</td>
</tr>
<tr>
<td>TDR² probe</td>
<td>7</td>
<td>Tensiometer</td>
<td>17</td>
</tr>
<tr>
<td>Tensiometer</td>
<td>10</td>
<td>Tensiometer</td>
<td>24</td>
</tr>
<tr>
<td>Suction lysimeter</td>
<td>13</td>
<td>Tensiometer</td>
<td>34</td>
</tr>
<tr>
<td>TDR probe</td>
<td>16</td>
<td>Saprolite-sand interface</td>
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<tr>
<td>Tensiometer</td>
<td>19</td>
<td>Base of sand</td>
<td>218</td>
</tr>
<tr>
<td>TDR probe</td>
<td>22</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Suction lysimeter</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Tensiometer</td>
<td>28</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TDR probe</td>
<td>31</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ceramic plate</td>
<td>38</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

† Time domain reflectometer.

Table 3. Irrigation schedules for three unsaturated saprolite columns.

<table>
<thead>
<tr>
<th>Column</th>
<th>Cycle</th>
<th>Irrigation duration‡</th>
<th>Irrigation interval</th>
<th>Irrigations per cycle</th>
<th>Cycle duration</th>
<th>Mean flux³</th>
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<tr>
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<td>120</td>
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<td></td>
<td>2</td>
<td></td>
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</tbody>
</table>

‡ Irrigation applied at rate of 0.6 cm² s⁻¹.

§ Mean flux is volume of injected water divided by the column cross sectional area (707 cm²) and time between cycles (12, 21.67, and 18 h for Columns 1, 2, and 3, respectively.
Calcium chloride was applied as a tracer to the top of each column through the irrigation system. A tracer solution containing 160 mg L\(^{-1}\) of calcium chloride was applied for 1 d on Columns 1 and 2 once steady flow conditions were obtained. Flush water containing a dilute solution of calcium nitrate was applied following each tracer injection experiment until measured outflow chloride concentrations returned to background. Pore waters were sampled using two high-flow, one-bar, ceramic lysimeters, measuring 1.27 cm in diameter by 27.94 cm in length (Soilmoisture Equipment Corporation). Lysimeters were maintained at a tension of 130 cm, enabling sufficient flow through the lysimeter to provide continuous collection of tracer data. Lysimeters were installed horizontally at depths of 13 and 25 cm below the surface in Columns 1 and 2 (Table 2). A series of traps and solenoid valves were used to route sampled waters from the lysimeter to the Cl\(^{-}\) analysis unit, while still maintaining a lysimeter tension of 130 cm. Pore waters were also sampled from porous plate effluent at the base of Column 2.

Pore waters were analyzed for Cl\(^{-}\) using a flow-injection, spectrophotometric method (Clinch et al., 1987; Holden et al., 1995; Baldwin, 1997). Mercuric thiocyanate and ferric nitrate reagents react with the Cl\(^{-}\) ions to form ferric thiocyanate, which is readily measured by colorimetric methods at 480 nm. Calibration was conducted for the range of 5 to 150 mg L\(^{-1}\) CaCl\(_2\). Output from the spectrophotometer was routed to a desktop computer.

**RESULTS AND DISCUSSION**

Summary physical and hydraulic data for four intact unsaturated saprolite columns are presented in Table 3. These data indicate that the saprolite studied here is dominated by the sand-sized fraction (66%), with little clay (13%), and a low bulk density (1.25 ± 0.02 g cm\(^{-3}\)). A porosity of 53% is estimated by assuming a grain size density of 2.65 g cm\(^{-3}\). Saturated hydraulic conductivities estimated using intact saprolite columns (27.3 ± 3.2 cm d\(^{-1}\)) compare favorably with field estimates (25.1 ± 3.3 cm d\(^{-1}\)).

Water retention data were used to estimate unsaturated model parameters for the Brooks–Corey, van Genuchten–Mualem, and Broadbridge–White formulations. The Galileo number relationship could not be fit due to the lack of unsaturated hydraulic conductivity data. The fit between observed and predicted water retention data appears good (Fig. 1). Predicted relative unsaturated hydraulic conductivity, \(K(\Theta)/K_s\), vs. relative saturations, \(\Theta\), are substantially different from each other (Fig. 1). While the Broadbridge–White and Galileo formulations are similar, both the Brooks–Corey and van Genuchten–Mualem formulations diverge rapidly.

One data point is available for comparison with the hydraulic conductivity predictions provided by the four models. The mean column flux (0.23 cm d\(^{-1}\)) obtained by averaging all irrigations in Columns 1 and 2, can be coupled with an assumed unit vertical hydraulic gradient to provide a relative hydraulic conductivity of 0.0085. This value is plotted against the relative saturation provided by the time domain reflectometry probes. Time domain reflectometry readings during the column spray experiments show steady volumetric water contents of 1.
Fig. 2. Cumulative normalized Cl\textsuperscript{−} breakthrough curves for unsaturated saprolite at the column surface and selected depths. Curve is normalized by dividing observed cumulative Cl\textsuperscript{−} mass by the total observed Cl\textsuperscript{−} mass. Tracer travel times, velocities, and effective porosities are provided in Table 5.

\[ \Theta = 0.44 \pm 0.01 \text{ cm}^3 \text{ cm}^{-3} \]

corresponding with a relative saturation of \( \Theta = 0.83 \). No substantial water content changes were observed in response to individual irrigations. The position of the single saturation–hydraulic conductivity pair indicates that the Broadbridge–White and Galileo formulations substantially overpredict the hydraulic conductivity in unsaturated saprolite, while the Brooks–Corey and van Genuchten–Mualem formulations provide better estimates. However, this interpretation is subject to uncertainty because the relative saturation estimated using the time domain reflectometry readings (0.83) is substantially less than obtained from water retention data at an approximate soil water pressure head of \(-100 \text{ cm (0.63)}\). Plotting the relative hydraulic conductivity at a different relative saturation clearly alters the interpretation, indicating that no clear statement of unsaturated hydraulic conductivity function accuracy can be made. It is also not possible to provide credible predictions of unsaturated hydraulic conductivity without additional field or laboratory information. Additional water retention data closer to saturation would reduce uncertainties in the unsaturated hydraulic conductivity function.

Chloride tracer velocities (Fig. 2, Table 4) are calculated based on the elapsed time between the median injection mass and the median observed tracer mass at each depth. Tracer velocities are slightly different between Columns 1 and 2, with faster velocities in Column 1 (6.00 and 3.13 cm d\textsuperscript{-1} at 13 and 25 cm, respectively) than at the same depths in Column 2 (3.72 and 2.53 cm d\textsuperscript{-1} at 13 and 25 cm, respectively). Differences between columns is not unexpected due to material heterogeneity. Tracer velocities decrease with increasing depth in both columns. Effective tracer porosity (Table 4), calculated as the ratio between the darcian flux (equal to 0.229 and 0.221 cm d\textsuperscript{-1} in Columns 1 and 2, respectively) and the tracer velocity, ranges between 3.8 and 7.3% in Column 1, and between 5.9 and 10.8% in Column 2. Effective porosities increase with increasing depth, and are substantially less than the water content of 44%. The anticipated mean tracer velocity for the known water content and flow rate is 0.53 cm d\textsuperscript{-1}. It is clear that some form of bypass or preferential flow is occurring, with an average velocity approximately four to 11 times faster than predicted using a volumetric water content of 44%. Rapid tracer velocities reduce contaminant transport arrival times and lead to errors in waste containment efficiency if not properly considered.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particle-size fractions</td>
<td>sand</td>
<td>0.66</td>
</tr>
<tr>
<td></td>
<td>silt</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>clay</td>
<td>0.13</td>
</tr>
<tr>
<td>Bulk density</td>
<td>( \rho_b )</td>
<td>1.25</td>
</tr>
<tr>
<td>Saturated water content</td>
<td>( \theta_s )</td>
<td>0.5283</td>
</tr>
<tr>
<td>Saturated hydraulic conductivity, field</td>
<td>( K_s )</td>
<td>25.1</td>
</tr>
<tr>
<td>Saturated hydraulic conductivity, lab</td>
<td>( K_s )</td>
<td>27.3</td>
</tr>
<tr>
<td>Brooks–Corey formulation</td>
<td>( \psi_b )</td>
<td>0.6465</td>
</tr>
<tr>
<td>van Genuchten–Mualem formulation</td>
<td>( \psi_m )</td>
<td>0.1430</td>
</tr>
<tr>
<td>Broadbridge–White formulation</td>
<td>( C )</td>
<td>10.30</td>
</tr>
<tr>
<td></td>
<td>( \psi_r )</td>
<td>52</td>
</tr>
<tr>
<td></td>
<td>( \theta_r )</td>
<td>0.2332</td>
</tr>
</tbody>
</table>

\( \dagger \) Estimated using an assumed grain density of 2.65 g cm\textsuperscript{-3}.

\( \ddagger \) Model parameters estimated using water retention data at soil water tensions of 50, 100, 200, 300, 400, and 500 cm.

\( \S \) Zero residual saturation.
Soil water pressure head observations are used to examine the dynamic response to irrigation events. Pressure head data obtained from tensiometer responses are similar in all three saprolite columns (Fig. 3 and 4). Pressure heads fluctuated rapidly; on the order of 15 to 20 cm in response to individual spray events, and 30 to 40 cm in response to changes in spray cycles. Pressure heads are between −80 and −120 cm at a depth of 7 cm in Columns 1 and 2, between −110 and −130 cm at 19 cm, and between −70 and −130 cm at all depths in Column 3. Observed fluid pressure heads are inconsistent with macropore flow because pores saturated within this range of pressure heads are substantially smaller than those associated with macropores (Beven and Ger-
Fig. 5. Observed and hydraulic advection–diffusion equation fit to soil water pressure head changes for unsaturated saprolite, Column 3, resulting from periodic irrigation at the column surface.

mann, 1982). However, our inability to measure fluid pressure head in all pores may limit our ability to infer flow through isolated macropores. Wave travel times at four depths (Fig. 5, Table 5) are obtained from soil water pressure head data in Column 3, during Cycle B5. Cycle B5 had a spray duration of 1 s followed by a 2-h interval before the next spray. Soil water pressure heads responded quickly at all depths, with increasing delay and attenuation with increasing depth (Table 6). Observed pressure wave velocities range between 1,983 cm d$^{-1}$ at 7 cm to 3670 cm d$^{-1}$ at 24 cm before slowing to 2695 cm d$^{-1}$ at 34 cm. The range in pressure wave velocities are approximately three orders of magnitude (1000 times) faster than observed tracer velocities of 2.04 to 6.00 cm d$^{-1}$. This large ratio may be underestimated because the mean fluid flux for Column 3 (0.071 cm d$^{-1}$) is approximately one-third of Columns 1 and 2 (0.229 and 0.221 cm d$^{-1}$, respectively). Wave travel times for Columns 1 and 2 were not determined because they were measured with substantially less accuracy than for Column 3.

Predicted kinematic ratios, $k$, as a function of relative saturation falls into two general groups (Fig. 1). Brooks–Corey and van Genuchten–Mualem model predictions are similar for the medium and dry range of saturations, only diverging near saturation. These models predict the kinematic ratio to be approximately 15 times faster than the fluid velocity for most water contents. Broadbridge–White and Galileo number models predict much smaller kinematic ratios for the saprolite column, with ratios of two and three, respectively, across the entire range. Regardless, these wave travel times are clearly many orders-of-magnitude less than those observed.

To better understand the reason for rapid pressure waves, water pressure head responses are fitted (Fig. 5) using the Leij–Toride solution, Eq. [16], of the advection–diffusion equation for a Dirac (spike) input. The qualitative resemblance between the observed and fitted shapes is good. The optimum fit was achieved by setting the hydraulic Peclet number, $k$, equal to zero, which is consistent with diffusion-dominated flow; that is, $D > c$ at shallow depths near the source. It is clear that it is inappropriate to apply the traditional wave velocity definition, $c = dK/d\theta$ because of the shallow

Table 5. Chloride tracer responses in two intact, unsaturated saprolite columns resulting from surface irrigation.

<table>
<thead>
<tr>
<th>Column</th>
<th>$z$</th>
<th>$t_p$</th>
<th>$v$</th>
<th>$n_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>cm</td>
<td>cm d$^{-1}$</td>
<td>cm d$^{-1}$</td>
<td>cm</td>
<td>%</td>
</tr>
<tr>
<td>1</td>
<td>13</td>
<td>2.17</td>
<td>6.00</td>
<td>3.8</td>
</tr>
<tr>
<td>25</td>
<td>7.99</td>
<td>3.13</td>
<td>7.3</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>13</td>
<td>3.50</td>
<td>3.72</td>
<td>5.9</td>
</tr>
<tr>
<td>25</td>
<td>9.89</td>
<td>2.53</td>
<td>8.7</td>
<td></td>
</tr>
<tr>
<td>38</td>
<td>18.64</td>
<td>2.04</td>
<td>10.8</td>
<td></td>
</tr>
</tbody>
</table>

$^\dagger$ Travel time ($t_p$) is interval required for half of observed tracer mass to pass the observation depth ($z$).

$^\ddagger$ Tracer velocity ($v$) is observation depth ($z$) divided by travel time.

$^\S$ Effective porosity ($n_e$) is darcian flux (0.229 and 0.221 cm d$^{-1}$ for Columns 1 and 2, respectively) divided by tracer velocity.

Table 6. Column 3 soil water pressure responses to periodic surface irrigation.

<table>
<thead>
<tr>
<th>$z$</th>
<th>$\psi_p$</th>
<th>$t_p$</th>
<th>$w$</th>
<th>$D_t$</th>
<th>$C_\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>cm</td>
<td>min</td>
<td>cm d$^{-1}$</td>
<td>cm$^2$ d$^{-1}$</td>
<td>cm$^{-1}$</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>15.28</td>
<td>5.08</td>
<td>1983</td>
<td>2314</td>
<td>3.07 $\times 10^{-7}$</td>
</tr>
<tr>
<td>17</td>
<td>9.91</td>
<td>6.75</td>
<td>627</td>
<td>10276</td>
<td>6.91 $\times 10^{-6}$</td>
</tr>
<tr>
<td>24</td>
<td>5.94</td>
<td>9.42</td>
<td>3670</td>
<td>14680</td>
<td>4.84 $\times 10^{-6}$</td>
</tr>
<tr>
<td>34</td>
<td>2.35</td>
<td>18.17</td>
<td>2695</td>
<td>15272</td>
<td>4.65 $\times 10^{-6}$</td>
</tr>
</tbody>
</table>

$^\dagger$ Wave velocity ($w$) is depth ($z$) divided by the time ($t_p$) of the maximum soil water pressure response ($\psi_p$).

$^\ddagger$ Hydraulic diffusion coefficient ($D_t$) determined using $D_t = t_p \left( w^2 - c^2 \right)/6$, where the best fit is obtained when $c = 0$.

$^\S$ Specific water capacity ($C_\delta$) determined using $C_\delta = K/D_t$, where $K = q = 0.071$ cm d$^{-1}$. 

nature of our experiment. However, the diffusion component becomes less important under field conditions as the depth of the unsaturated zone increases, and the traditional wave velocity applies.

Calculated hydraulic diffusivity coefficients are large, ranging from 2314 to 15 272 cm$^2$ d$^{-1}$, with an increasing trend observed with depth. We can also calculate the specific water capacity, \( C_p = \frac{\partial h}{\partial \theta} = K/D \), by assuming that the hydraulic conductivity equals the mean flux of water (4.5 \times 10^{-4} cm$^2$). The observed specific water capacities range from 4.65 \times 10^{-4} to 30.7 \times 10^{-4} cm$^{-1}$), which is two orders of magnitude greater than the compressibility of water (4.5 \times 10^{-8} cm$^{-3}$). Calculated specific water capacities are much lower, however, than the anticipated value (5 \times 10^{-4} cm$^{-1}$) calculated from the water retention curve at the ambient soil water pressure head (−100 cm). Small specific water capacity values are consistent with water content observations that show no detectible change as the pressure wave passes. Apparently, the water saturation change is intermediate (in a logarithmic sense) between an incompressible fluid and a response in which the water content change has sufficient time to equilibrate with the ambient soil water pressure head. Hysteresis in the water retention function may account for the small specific water capacity values. We are unable to explain why hydraulic diffusivity rates increase, and specific water capacity coefficients decrease, with depth.

**CONCLUSIONS**

Unsaturated hydraulic and transport properties of intact saprolite columns were obtained using short-duration irrigation at the upper surface. By limiting the duration and frequency of irrigations, soil water pressure heads were maintained between approximately −70 and −130 cm, and the relative hydraulic conductivity was reduced to 0.0085 at a relative saturation of 0.83. This low hydraulic conductivity value precluded the use of conventional steady flow control. Tracer properties were obtained by adding a Cl$^-$ tracer to the irrigation pulses. Rapid tracer movement (2.04 and 6.00 cm d$^{-1}$) is consistent with preferential flow through an active porosity (0.038 and 0.108 cm$^3$ cm$^{-3}$) substantially less than the ambient water content (0.44 cm$^3$ cm$^{-3}$). Use of short-duration irrigation pulses thus provides a viable means for estimating unsaturated hydraulic and transport properties under laboratory, and perhaps field, conditions.

An important discovery was that soil water pressure wave velocities (1983–3670 cm d$^{-1}$) were approximately 1000 times faster than the average Cl$^-$ tracer velocity. Kinematic theory is one possible reason for rapid fluid pressure head responses. We show, using kinematic theory applied to four parametric models (Brooks–Corey, van Genuchten–Mualem, Broadbridge–White, and the Galileo number), that rapid hydraulic responses should be expected in the unsaturated zone for small inputs. Kinematic wave velocities are predicted to be faster than tracer velocities regardless of the type of unsaturated hydraulic conductivity model used. The four models predict pressure wave travel times of between two and fifteen times the tracer velocity.

Observed pressure wave velocities are substantially faster than those predicted based on kinematic theory alone. Instead, a kinematic form of Richards' equation is derived that is mathematically identical to the advection–diffusion equation used in solute transport. The solution of the hydraulic advection–diffusion equation for a dirac (spike) input provides an excellent fit of the observed hydraulic response when the hydraulic Peclet number is zero. This result indicates that rapid pressure head responses result from large hydraulic diffusivities. Because the hydraulic Peclet number should increase with depth, we speculate that pressure wave velocities should decrease with increasing depths, yielding a pressure wave velocity consistent with kinematic theory at greater depths. This approach provides an opportunity to estimate unsaturated hydraulic properties using pressure wave travel times.

One concern generated by our experiments is the potential for incorrect estimation of hydraulic and transport parameters. Rapid travel times of fluid pressure head or water content through the unsaturated zone due to transient testing could be mistakenly interpreted as preferential flow, or as large hydraulic conductivities. The experimental results presented here show that large hydraulic diffusivities could cause rapid pressure wave movement in homogeneous, unsaturated media. Rapid hydraulic responses arise from the translation and dispersion of a pressure, or energy, wave. This concern is particularly acute when laboratory or field tests confuse the Richards' equation response with either the darcian flux, fluid velocity, or kinematic response. Tracer testing may be preferable for estimating preferential flow properties, while steady flow testing may provide less ambiguous estimates of unsaturated hydraulic conductivity.

**REFERENCES**


