Chapter 6

Ground-Water Hydrology

The subsurface is invisible and, hence, mysterious. Stories abound about subterranean worlds, underground rivers and lakes, and lost civilizations. Because access to the subsurface is limited to caves, mines, and other small openings, few people have ever visited this Stygian realm.

In this chapter we focus on how water flows through the subsurface. We start with a general description of the physical environments that exist in the subsurface, move on to how water flows through these environments, and conclude with issues related ground-water quality.

6.1 Subsurface Features

Water flow is a function of two factors:

- the physical structure of the subsurface with respect to the size and arrangement of voids, especially the presence or absence of aquifers, fractures, and other features; and
- the availability of water and the resulting fluid pressures that develop.

The first factor is quantified using a coefficient called the *hydraulic conductivity*, which is a measure of the permeability of the geologic unit. The second factor is represented using the *hydraulic gradient*, which is a measure of the force driving water through the geologic structure.

Because water flows downward due to gravity, a zone of *saturation* - or *phreatic* zone - develops within the subsurface. Fluid pressures are positive within the zone of saturation, with the pressure increasing as the depth below the water surface increases. Water flow in the saturated zone is normally close to horizontal in *aquifers* - zones of higher permeability - and is normally close to vertical in *aquitards*, also called *confining layers* - zones of lower permeability.

Above the zone of saturation lies the *unsaturated* - or *vadose* - zone, where water is bound by capillary forces to the soil. These forces are the result of adsorptive, or surface tension, forces that attract water to soil surfaces, and result in negative pressures. The unsaturated zone normally extends from the ground surface down to the *water table*, which is the surface that separates the saturated and unsaturated zones.

Water flow in the unsaturated zone is normally close to vertical due to the downward force of gravity, switching over to horizontal flow once it reaches the water table.

Zones of *perched water* may exist within the unsaturated zone. Perched water results from the accumulation of downward percolating water on top of layers of lower permeability. Water flow in perched layers is more horizontal than in the rest of the unsaturated zone.

The *surficial* aquifer is the uppermost saturated unit below the water table. The surficial aquifer extends from the water table downward to a confining layer, below which lies one or more *confined* aquifers. Changes in fluid pressure within surficial aquifers results in changes in the upper surface, i.e., the water table, and *vice versa*.

Confined aquifers are sandwiched above and below by confining layers, and acts much as a pipe does. Changes in fluid pressure does not affect the position of the confining layer, and thus water in confined aquifer acts more like an incompressible fluid.

Fluid Head

Water moves from high head to low head. The forces of gravity, pressure, and inertia can be combined to yield a general equation of the energy status of water, described by Bernoulli's equation:

$$h = z + \frac{P}{\gamma} + \frac{v^2}{2\,g} \tag{6.1}$$

where h is the total head, z is the elevation, P is the fluid pressure, $\gamma = \rho\, g$ is the fluid weight, ρ is the fluid density, g is the gravitational constant, and v is the fluid velocity.

The total head is the variable used to predict the direction and magnitude of fluid flow, in both surface water and groundwater systems. The velocity (inertial) component can generally be neglected if the flow is slow, especially true in groundwater systems, $v \ll 0$. When these inertial effects in ground-water systems can be neglected, then the total head, h (m), can be written using:

$$h = z + \frac{P}{\gamma} = z + p \tag{6.2}$$

where $p = P/\gamma$ (m) is the fluid pressure head. Pressure changes with depth and time can result from barometric (atmospheric pressure) influences, tidal effects, fluid density (sediment, salinity) changes, and vertical flow (non-hydrostatic conditions) within the water column.

The pressure term can also be neglected by measuring the water surface elevation, $h = z$, which is where the fluid pressure equals zero, $p = 0$. This simplification requires hydrostatic conditions, within the monitoring device (e.g., borehole, piezometer) i.e.:

$$\Delta p = -\gamma \; \Delta z \tag{6.3}$$

which implies that a decrease in elevation, Δz, is accompanied by a corresponding increase in pressure, Δp, at a rate specified by the fluid weight, γ.

Thus, the water surface elevation is only an appropriate measure of the total head of the system, $h \approx z$, given that the velocity of the water is sufficiently small, pressure changes with depth and time can be neglected, and free water is present.

Capillary and Osmotic Forces. Capillary and osmotic forces can also induce fluid movement and thereby affect the total head. Failure to account for these forces may result in incorrect predictions of water flow and transport.

Capillary forces arise due to the tendency of soil materials to absorb and bind water to soil surfaces. The water bound to soil surfaces resists the downward force of gravity, and does not readily drain from the soil. The total head must consider the negative fluid pressures that arise due to these *absorbtive* forces (also call matric tension). Matric tensions are commonly measured using *tensiometers*.

Water can move upward above the regional water table due to capillary forces. Finer grained media have greater capillary forces, which result in higher capillary fringes. The height of the saturated zone formed above the water table (i.e., the capillary fringe) that forms is largely determined by the magnitude of capillary forces, which is a function of the pore surface area. Capillary forces generally increase with decreasing pore size. The capillary rise equation commonly relates the pore size to the height of rise:

$$\psi = \frac{2\,\sigma \cos\alpha}{r\,\gamma} \tag{6.4}$$

where ψ is the capillary height of rise, σ is the surface tension of water, α is the solid-liquid contact angle, and r is the pore radius (Hillel, 1971).

An additional force that induces fluid movement is a change in solute concentration. The osmotic potential increases the total head of water, causing water to flow from areas of low solute concentrations to areas where solute concentrations are higher.

In arid areas with high soil solute concentrations and high evapotranspiration rates, solutes may become concentrated as the water evapotranspires, causing local increases in the surficial solute concentration. This surficial increase in solute concentration induces an additional force that causes water to move upward from less saline groundwater to the surface.

Thus, evapotranspiration can increase the height of the capillary fringe by augmenting the capillary force with osmotic forces. The osmotic height of rise in dilute solutions is:

$$\phi = \frac{k\;T}{\gamma}\;C \tag{6.5}$$

where ϕ is the osmotic height of rise, k is the Boltzmann constant, T is the absolute temperature, and C is the solute concentration (Hillel, 1971).

Darcy's Law

Darcy's law is used to predict the rate of flow through geologic media. The water flow rate is written as a *flux*, or volume of water per unit area of cross-sectional area per unit time:

$$\vec{q} = \frac{Q}{A} = -K\;\nabla h \tag{6.6}$$

where

- $\vec{q}$ is the flux vector
- Q is the volume of water flowing per unit time
- A is the cross-sectional area through which the water flows
- K is the hydraulic conductivity of the geologic media
- ∇h is the hydraulic gradient, or slope of the total head

Hydraulic Gradient. The hydraulic gradient is the change in total head with distance, or:

$$\nabla h = \frac{\partial h}{\partial x} + \frac{\partial h}{\partial y} + \frac{\partial h}{\partial z} \tag{6.7}$$

Note that the hydraulic gradient is a vector, composed of three parts, pointing in the direction of maximum change of the total head surface.

The total fluid flow, Q (m^3/s), is the product of the fluid flux and the cross-sectional area, A (m^2), perpendicular to flow:

$$Q = q\;A \tag{6.8}$$

Hydraulic Conductivity. The aquifer hydraulic conductivity, K (m/s), is the property which describes the ability of water to flow through porous media. The hydraulic conductivity can be decomposed into three terms:

$$K = \frac{\gamma}{\mu}\,k \tag{6.9}$$

where k (m^2) is the intrinsic permeability, γ (Pa/m) is the fluid specific weight, and μ (Pa s) is the fluid dynamic viscosity. In the petroleum industry, Darcy's law is often written as:

$$\vec{q} = -\frac{k}{\mu}\ \nabla\phi \tag{6.10}$$

where $\phi = \gamma\, h$ (Pa) is the total potential. The important difference is the choice of the reference fluid specific weight, which depends entirely upon which fluid one uses as the manometer fluid. In most hydrology applications, the *freshwater density* is used, i.e., $\rho = 1\ kg/L$, so that the resulting total head is referred to as the *freshwater head*.

The intrinsic permeability, k, is related to pore size of granular media using:

$$k = Cd^2 \tag{6.11}$$

where C is a constant and d is the average pore diameter. For fractured media, we have:

$$k = \frac{e^2}{2} \tag{6.12}$$

where e is the *fracture aperture*, or fracture opening width. Note that the permeability increases with the square of the average pore diameter or fracture aperture.

The permeability of an aquitard is usually distinguished from that of the aquifer using a *prime* to distinguish them from aquifer properties. For example, K' and b' describe the hydraulic conductivity and thickness of the confining layer.

The permeability may be spatially uniform, so that the hydraulic conductivity can be represented by a constant. Such geologic units are called *homogeneous*, while units that vary spatially are termed *heterogeneous*.

If the permeability is the same in all directions, we say that the aquifer is *isotropic*. In some cases, the permeability in one direction does not equal that in other directions. In these cases, we say that the aquifer is *anisotropic*, and the flux is defined using:

$$\vec{q} = -\underline{\underline{K}}\ \nabla h \tag{6.13}$$

where

$$\underline{\underline{K}} = \begin{bmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{bmatrix} \tag{6.14}$$

is a tensor that *rotates* the flow from the direction of the hydraulic gradient. Thus, if the hydraulic gradient is steep in one direction, but a more preferred path is available in a different direction, then the flow direction is altered because of the more favorable permeability in that direction.

In many cases, the horizontal components are equivalent, $K = K_x = K_y$, but the vertical hydraulic conductivity, $K' = K_z$, is substantially less than the horizontal components. In these cases, an anisotropy ratio can be defined as:

$$m = \frac{K}{K'} \tag{6.15}$$

Drawdown - Water level decline due to pumping from well, relative to the baseline (initial) water level before pumping.

Hydraulic Gradient - Change in total head per unit distance

Flux, or Darcian Velocity - The volume of water flowing per unit area per unit time

Darcy's Law - A relationship between water flux and the product of the hydraulic gradient and the hydraulic conductivity

Total Head - Sum of elevation, pressure, velocity, osmotic and other potentials

Total Flow - The cumulative flow over an area, calculated using the product of the flux with the cross-sectional area of flow.

Fluid Velocity - The rate at which fluid particles are moving, equal to the darcian velocity divided by the effective porosity.

Transmissivity - The product of the hydraulic conductivity and the aquifer thickness, equal to the integral of the hydraulic conductivity over the vertical extent of the aquifer.

Specific Yield - The volume of water released from storage from an unconfined aquifer per unit volume of aquifer per unit decline in watertable elevation.

Specific Storage - A measure of the volume of water released (or added) to storage, per unit volume of aquifer, per unit increase (or decrease) in fluid pressure. It is the sum of the aquifer plus water compressibilities.

Storativity - A measure of the volume of water released (or added) to storage, per unit *area* of aquifer, per unit increase (or decrease) in fluid pressure.

Aquifer Compressibility - The ability of an aquifer to expand and contract. The reciprocal of bulk modulus of elasticity of aquifer. The compressibility ranges from $10^{-6}Pa^{-1}$ for clays, to $10^{-10}Pa^{-1}$ for rock.

Water Compressibility - The ability of water to expand and contract, equal to $4.4 \times 10^{-10}\ Pa^{-1}$.

Hydraulic Diffusivity - The ratio of the hydraulic conductivity to the specific storage, equal to the ratio of the transmissivity to the storativity.

Barometric Efficiency - The change in water levels in wells due to changes in barometric pressure.

Tidal Efficiency - The change in water levels in wells due to changes in ocean levels.

Aquifer Storage

In unconfined aquifers, the volume of water released from storage per unit area of aquifer per unit decline in the watertable, called the *specific yield*, S_y $(m^3/m^3/m)$, is similar in magnitude to the volumetric water content, θ. The observed drainage of water is generally less than the water content because of residual saturation within the media, θ_r, described in a later chapter, held by surface tension forces in the smaller pores.

$$S_y = b\,\frac{d\,\theta}{d\,h} \tag{6.16}$$

In confined aquifers, the volume of water released from storage per unit decline in pressure head, called the *specific storage*, S_s $(m^3/m^3/m)$, is much smaller than the specific yield. This is because the water is released due to elastic deformation of the water and the aquifer:

$$S_s = -\frac{d\,\theta}{d\,h} = -\frac{d\left[\frac{V_w}{V_b}\right]}{d\,h} \tag{6.17}$$

or:

$$\begin{aligned} S_s &= \frac{V_w}{V_b^2}\,\frac{d\,V_b}{d\,h} - \frac{1}{V_b}\,\frac{d\,V_w}{d\,h} \\ &= \frac{V_w}{V_b}\left[\frac{1}{V_b}\,\frac{d\,V_b}{d\,h} - \frac{1}{V_w}\,\frac{d\,V_w}{d\,h}\right] \end{aligned} \tag{6.18}$$

which is the same as:

$$S_s = \gamma\; n\;\left(\beta_w - \beta_s\right) = \gamma\;\left(\alpha + n\;\beta_w\right) \tag{6.19}$$

where α is the aquifer compressibility, β_w is the water compressibility, and n is the aquifer porosity. The water compressibility is generally defined using:

$$\beta_w = -\frac{1}{V_w}\,\frac{d\,V_w}{d\,\Phi} \tag{6.20}$$

and the aquifer compressibility is:

$$\alpha = -\frac{1}{V_b}\,\frac{d\,V_b}{d\,\Phi} \tag{6.21}$$

where Φ is the effective stress, V_b is the bulk volume, and β_s is the skeletal compressibility.

The equivalent volume of water released per unit *area* of aquifer per unit decline in pressure head is called the storativity, S $(m^3/m^2/m)$. The storativity is the total volume of water released over the thickness of the aquifer, b, so that $S = b\,S_s$.

The hydraulic diffusivity is the ratio of the hydraulic conductivity to the specific storage, equal to the ratio of the transmissivity to the storativity:

$$D = \frac{K}{S_s} = \frac{T}{S} \tag{6.22}$$

As noted below, the hydraulic diffusivity is useful for describing the movement of a perturbation in fluid heads through an aquifer.

Dual Porosity A common feature of the subsurface is rapid movement through larger *macropores*, and slower fluid movement through *micropores*. The preferential flow of fluids through macropores tends to bypass the micropores. Over time, however, the solute slowly diffuses into the micropores until the concentration is the same in both porosities.

The dual porosity problem acts in reverse once the source of solutes is eliminated. Now, fresh water displaces the water in the macropores, but slow diffusion-limited processes cause the micropores to continuously *leak* solutes into the fresh water. Cleanup of contaminated sites is commonly limited by these slow diffusion rates.

Barometric Efficiency. One manifestation of the aquifer storage is the influence of barometric pressures on water levels in wells penetrating confined aquifers. It is commonly observed that water levels drop, $-\Delta WL$, as the barometric pressure head rises, ΔBP. This can be written as:

$$BE = -\frac{\Delta WL}{\Delta BP} = \frac{n}{S_s} \tag{6.23}$$

Note that the barometric efficiency, BE, of a confined aquifer is directly related to the ratio of the porosity to the specific storage.

Similar effects are observed when ocean tides cause surface water levels to change. In this case, we see an inverse effect to the barometric efficiency - increasing sealevels cause increasing water levels in wells. This is called the *tidal efficiency*, TE, and is related to the barometric efficiency through the relationship $BE + TE = 1$.

Leakage

The vertical leakage of water across an aquitard is called the aquitard flux, q' (m/s), and is determined using:

$$q' = -K'\frac{\Delta h}{b} = -\frac{K'}{b'}\Delta h = C\Delta h = \frac{\Delta h}{c} \tag{6.24}$$

where K' (m/s) is the vertical hydraulic conductivity of the aquitard, b' (m) is the aquitard thickness, Δh (m) is the total head difference across the aquitard, C (1/s) is the aquitard conductance, and c (s) is the hydraulic resistance.

Another term used to characterize aquitards is the leakance, L (m):

$$L = \sqrt{Tc} = \sqrt{(Kb)\left(\frac{b'}{K'}\right)} = \sqrt{mbb'} \tag{6.25}$$

where $m = K/K'$ is the anisotropy ratio.

Leakage into aquifers during aquifer tests often result in steady flow conditions. This attribute is quantified using the specific capacity coefficient, C_p:

$$C_p = \frac{Q}{s} \tag{6.26}$$

where Q (L/s) is the imposed pumping rate and s (m) is the equilibrium drawdown.

Aquitard Flux - The vertical movement of water across an aquitard.

Aquitard Conductance - Hydraulic conductivity of resistive layer per unit thickness of the resisting layer.

Hydraulic Resistance - The reciprocal of the aquitard conductance.

Aquitard Leakance - The product of the aquifer transmissivity and the hydraulic resistance. A measure of the relative contribution of horizontal flow through an aquifer to the vertical leakage into an aquifer.]

Specific Capacity Water discharge from well divided by the resulting drawdown in the pumped well.

Tortuosity

Tortuosity also results in spreading of a solute. Tortuosity is defined as the ratio of the path length to the ruler (straight-line) length.

$$\tau = \frac{\Delta s}{\Delta x} \tag{6.27}$$

where τ is the tortuosity, Δs is the path length, and Δx is the ruler (or straightline) length.

Longer path lengths means that the solute spreads out over larger areas than it would normally if it were to move in a straight line between two points. The path length along a streamline between two points is generally unknown, however. In general, the parameters which may be measured using experimental tests are:

- the effective porosity of the geologic material, n
- the total head difference, Δh, over a specified distance, Δx
- the hydraulic gradient, $i = \Delta h/\Delta x$, over a specified distance, Δx
- the hydraulic conductivity, K, over a specified distance, Δx
- the travel time, $t_t = \Delta x/\Delta t$, over a specified distance, Δx

The hydraulic gradient can be calculated at two scales, as a straightline gradient along Δx, or along the curve associated with the true path described by Δs

$$i_x = \frac{\Delta h}{\Delta x} \tag{6.28}$$

$$i_s = \frac{\Delta h}{\Delta s} \tag{6.29}$$

It is easy to see that

$$i_x = \frac{\Delta h}{\Delta x} = \frac{\Delta h}{\Delta s}\ \frac{\Delta s}{\Delta x} = i_s\tau \tag{6.30}$$

or

$$i_s = \frac{i_x}{\tau} \tag{6.31}$$

The hydraulic conductivity at the experimental scale can also be related to the value at the streamline scale:

$$K_x = \frac{q}{i_x} = \frac{q}{\tau i_s} = \frac{K_s}{\tau} \tag{6.32}$$

$$K_s = \frac{q}{i_s} = \tau\ \frac{q}{i_x} = \tau K_x \tag{6.33}$$

For extrapolating tests from one scale, say at a field or laboratory scale of size, Δx_1, to a different scale, Δx_2, the following relationship can be used:

$$\frac{K_1}{K_2} = \frac{i_2}{i_1} = \frac{\tau_2}{\tau_1} \tag{6.34}$$

Similar to the gradient and the hydraulic conductivity, the calculated travel time may also be affected by the scale of measurement. The travel time is defined here as the integral of the inverse velocity along a one-dimensional streamline:

$$t_t = \int_{s_a}^{s^b} v^{-1} ds \tag{6.35}$$

where t_t is the fluid travel time, v is the fluid velocity along streamline, s is the distance along streamline, and s_a and s_b are the particle starting and ending positions, respectively. The fluid velocity is the volumetric flow rate per unit area (i.e., the darcian flux) divided by the porosity, or:

$$v = \frac{q}{n} = \frac{Q}{nA} = -\frac{K_s i_s}{n} \tag{6.36}$$

where $q = -K_s i_s$ is the fluid flux, n is the porosity, Q is the total flow, A is the cross-sectional area, K_s is the local hydraulic conductivity, and i_s is the local hydraulic gradient. By assuming constant velocity along the streamline, we obtain:

$$t_t = \frac{\Delta s}{v} = \frac{n\Delta s}{q} = -\frac{n\Delta s}{Ki} = -\frac{n\Delta s^2}{K_s\Delta h} \tag{6.37}$$

where $i = \Delta h/\Delta s$, and where $\Delta s = s_b - s_a$ is the distance along the streamline, and $\Delta h = h_b - h_a$ is the head drop along the streamline. Switching to ruler lengths, we have:

$$t_t = -\frac{n\tau^2\Delta x^2}{\tau K_x\Delta h} = -\frac{n\tau\Delta x^2}{K_x\Delta h} \tag{6.38}$$

If the concept of fractal scaling is employed, then a relationship between the tortuosity at one scale can be related to the tortuosity at a different scale:

$$\tau_1 = \tau_o^{\beta\Delta x_1} \tag{6.39}$$

and

$$\tau_2 = \tau_o^{\beta\Delta x_2} \tag{6.40}$$

where β is a fractal scaling parameter and τ_o is a dimensionless fractal tortuosity parameter.

Formation Factor An effect of scale on measurements was noted in early geophysical measurements using electrical resistivity. In this case, the bulk resistivity of the medium changed with scale. In an effort to predict this change in scale, Archie used the following equation:

$$F = R_o / R_w \tag{6.41}$$

where F is the *formation factor*, R_o is the bulk resistivity of the porous medium and R_w is the bulk resistivity of fluid. If $R_o = RL_e/A_e$ and $R_w = RL/A$, then

$$F = \frac{\frac{L_e}{L}}{\frac{A_e}{A}} = \frac{1}{\tau n} \tag{6.42}$$

where R is the pure fluid resistivity, $n = A_e/A_T$ is porosity and τ is the tortuosity.

Comments This section demonstrates the effect of tortuosity and the scale of measurement on the hydraulic conductivity, hydraulic gradient, and travel time. A critical parameter in this analysis is the tortuosity, a result of the geometry of the flow regime. Additional research is required related to the effects of spatial variability on tortuosity, and the relationship between scale and the estimated tortuosity.

The spatial variability of tortuosity along the streamline may or may not have significant effects on the estimated travel time. It may be possible that local fluctuations in this parameter may not significantly affect regional travel times. Also, if the tortuosity is scale invariant, then laboratory and field parameter estimates can be directly applied to regional-scale models without the need for incorporating scale effects.

Problems

1. Two monitoring wells are located immediately next to each other. The two wells are *completed* (open) at different depths below the surface. The shallower well is perforated between 90 and 100 feet below the surface, while the deeper well is perforated between 190 and 200 feet. The rest of the well casing is *blank* (solid). The water level in the shallow well is four feet below the water level in the deeper well.
   (a) Explain how the water levels in two wells could be so different.
   (b) Calculate the direction and magnitude of the vertical hydraulic gradient.
   (c) Calculate the vertical flux if the vertical hydraulic conductivity is $K_z = 0.1\ ft/day$
   (d) Calculate the vertical movement of water over a one square mile area.

2. Water levels in three wells completed in the same aquifer have been measured:

Well	x	y	h
1	0	0	0
2	100	0	30
3	0	100	50

   (a) What is the hydraulic gradient in the x- and y-directions?
   (b) What is the *magnitude* and *direction* of the hydraulic gradient?
   (c) The hydraulic conductivity is measured to be $K = 10\ cm/day$
   - What is the flux in the x- and $y-$directions?
   - What is the *magnitude* and *direction* of the flux?

   (d) The effective porosity is measured to be $n = 0.30$
   - What is the fluid velocity in the x- and $y-$directions?
   - What is the *magnitude* and *direction* of the fluid velocity?
   - What is the *travel time* for a distance of $L = 100\ m$?
   - What is the *travel time* if the *tortuosity* is $\tau = 1.4$?

6.2 Governing Equations

In this section, we present the equations that control the movement of water and any solutes carried by the water. These general equations are written as partial differential equations, i.e., equations that relate a variable - such as hydraulic head, h, or solute concentrations, C - as a function of space, (x, y, z), and time, t.

Fluid Flow

A conservation equation can be written that defines how water flows through the subsurface:

$$\nabla \cdot \vec{q} = -\frac{\partial \theta}{\partial t} \tag{6.43}$$

This equation states that a change in flux, i.e., outflows minus inflows, must be balanced by a loss or accumulation of water. That is, if the outflows exceed the inflows per unit distance, then there will be a loss of water storage in the system.

We can make the following substitutions:

Darcy's Law $\vec{q} = -\ K\ \nabla h$

Storage Coefficient $S_s = \partial\theta/\partial h$

so that we have:

$$K\ \nabla^2 h = S_s\ \frac{\partial h}{\partial t} \tag{6.44}$$

where $\nabla^2 h = \nabla \cdot \nabla h$. This equation is the same as:

$$D \; \nabla^2 h = \frac{\partial h}{\partial t} \tag{6.45}$$

where $D = K/S_s$ is the hydraulic diffusivity.

Finite Difference Approximation. We can also write this as a *finite difference* equation by approximating continuous changes with incremental changes. In one dimension, this is equivalent to:

$$\frac{\partial^2 h}{\partial x^2} \approx \frac{\Delta\left[\frac{\Delta h}{\Delta x}\right]}{\Delta x} \qquad \textit{and} \qquad \frac{\partial h}{\partial t} \approx \frac{\Delta h}{\Delta t} \tag{6.46}$$

substitution yields:

$$D \; \frac{\frac{h(i+1,j)-h(i,j)}{\Delta x} - \frac{h(i,j)-h(i-1,j)}{\Delta x}}{\Delta x} = \frac{h(i,j+1)-h(i,j)}{\Delta t} \tag{6.47}$$

where $h(i,j)$ is the total head at point i at time j. This equation is equivalent to:

$$\frac{D \; \Delta t}{\Delta x^2} \; \{[h(i+1,j)-h(i,j)]-[h(i,j)-h(i,j-1)]\} = h(i,j+1)-h(i,j) \tag{6.48}$$

which can be re-arranged to yield:

$$h(i,j+1) = [1-2\,\beta]\; h(i,j) + \beta\,[h(i+1,j)+h(i-1,j)] \tag{6.49}$$

where $\beta = D \; \Delta t/\Delta x^2$. This equation predicts the total head at every node for the next time step based upon the head at the node and the two neighboring nodes at the current time. This is called a *forward* difference equation, because we use current observations to predict the future head.

Care must be taken in selecting time steps, Δt, and spatial increments, Δx, so that the value of α does not grow too large. Values of α greater than one-half are sure to cause numerical instabilities in the solution.

Two-Dimensional Flow The same approach can be used in two-dimensions, yielding:

$$h(i,j,k+1) = [1-4\;\beta]\; h(i,j,k) + \beta\,[h(i+1,j,k)+h(i-1,j,k)+h(i,j+1,k)+h(i,j-1,k)] \tag{6.50}$$

where $h(i,j,k)$ is the head at x-location i, y-location j, and time k, and where $\Delta x = \Delta y$ and $\beta = D \; \Delta t/\Delta x^2$ is the same as before.

Steady Flow The finite difference equations can be simplified for steady flow, shown in Table 6.1:

Boundary Conditions Note that values of hydraulic head must be specified along the boundaries of the flow domain - you have to start and stop somewhere. A *specified head* boundary condition means that the first and last node have a value assigned to them, i.e., $h(1) = h_1$ and $h(n) = h_n$ where h_1 and h_n must be given.

Alternatively, a *boundary flux* can also be specified. In this case we have:

$$q_1 = q(1) = -K \; \frac{h(2)-h(1)}{\Delta x} \tag{6.54}$$

so that we can specify the head at the boundary using:

$$h_1 = h(1) = h(2) + \frac{q_1}{K} \tag{6.55}$$

If a *no-flow boundary* is present, then $q_1 = 0$ and $h_1 = h(2)$.

Solution of these equations can be obtained using a simple spreadsheet. More complicated solutions for variable hydraulic conductivity, K, are also possible by explicitly incorporating nodal values, K_i in the equations.

Solute Transport

Water flowing through the subsurface carries with it dissolved and small particulate matter. This material is *advected* by the carrier fluid, and also *disperses* as it flows through the medium. The advection-dispersion equation, ADE, is an approximate expression that describes the general transport of ground-water solutes:

$$D \; \nabla^2 C - v \; \nabla C = \frac{\partial C}{\partial t} \tag{6.56}$$

where D is the solute dispersion coefficient, C is the solute concentration, and v is the advective transport velocity. The dispersion coefficient is a function of the diffusion rate of the solute in still water, D_o, and the spreading induced by multiple flowpaths and non-uniform velocities:

$$D = D_o + \alpha \; v \tag{6.57}$$

where α is the solute dispersivity, which tends to increase with distance, and accounts for the spreading of a solute as the fluid velocity increases. A well-mixed system has a large dispersivity, while a poorly-mixed system as a small dispersivity.

Peclet Number The Peclet number, P_e, is the ratio of advective to dispersive forces:

$$P_e = \frac{v \; L}{D} \tag{6.58}$$

where v is the fluid velocity, L is a length scale, and D is the solute dispersion coefficient. A large Peclet number implies that advection is much larger than dispersion, while a small Peclet number implies that dispersion dominates.

Table 6.1: Steady flow finite difference equations.

- One-Dimensional:

$$h(i) = \frac{h(i+1) + h(i-1)}{2} \tag{6.51}$$

- Two-Dimensional:

$$h(i,j) = \frac{h(i+1,j) + h(i-1,j) + h(i,j+1) + h(i,j-1)}{4} \tag{6.52}$$

- Three-Dimensional:

$$h(i,j,k) = \frac{h(i+1,j,k) + h(i-1,j,k) + h(i,j+1,k) + h(i,j-1,k) + h(i,j,k+1) + h(i,j,k-1)}{6} \tag{6.53}$$

If we note that the effective dispersion is a function of velocity, then we have:

$$P_e = \frac{v\ L}{D_o + \alpha\ v} \approx \frac{L}{\alpha} \approx 1 \tag{6.59}$$

where we neglect molecular diffusion, $D_o \approx 0$, and where we previously noted that the dispersivity is approximately equal to the length scale, $\alpha \approx L$. This implies that advective and dispersive forces are equally balanced.

Problems

1. Construct a ground-water flow model using a spreadsheet.
   (a) In Excel, go to Tools > Options > Calculation: Check the *Iteration* box
   (b) Enter the following boundary conditions:
      - Cell B1: Enter a value of 20. Copy across to Cell K1. *(Constant Head)*
      - Cell B21: Enter a value of 0. Copy across to Cell K21. *(Constant Head)*
      - Cell A1: Enter the equation $= B1$. Copy down to Cell A21. *(No Flow)*
      - Cell L1: Enter the equation $= K1$. Copy down to Cell L21. *(No Flow)*
   (c) Enter the following flow equations:
      - Cell B2: Enter the equation $= (A2 + C2 + B1 + B3)/4$. Copy down and across to Cell K20.
   (d) Iteratively solve the problem. Press the <F9> key until the numbers converge.
   (e) Plot the values of head in Cells A1 through L21 using the *surface* plot.
      - Select the data on the spreadsheet from Cell A1 to Cell L21.
      - Select the *graph icon* from the toolbar.
      - Select the *surface* option on the menu.
      - Select *finish.*
   (f) Change the value of head in Cell F6 to 0 to simulate a pumping well. Resolve and replot the values of head.

6.3 Example: Savannah River Site

The Savannah River Site (SRS) is located in the Atlantic Coastal Plain physiographic province, which extends from Cape Cod, Massachusetts, to south central Georgia. This province is underlain by seaward dipping unconsolidated and poorly consolidated sediments that increase from a thickness of zero at the Fall Line to more than 1,200 m (4000 feet) at the South Carolina Coast. The sediments of the Atlantic Coastal Plain were deposited under a variety of conditions and have formed a complex system of transmissive and confining units. The SRS is located near the updip edge of the Atlantic Coastal Plain sequence where the sedimentary wedge thins dramatically and undergoes abrupt facies changes (Aadland et al., 1995).

The Cretaceous and Cenozoic sediments which underlie the study area dip to the southeast at an average of 6.6 meters per kilometer (35 ft/mi). Sediments thicken from 200 m (600 ft) in the northwest part of SRS to 350 m (1,100 ft) near the center of SRS to more than 550 m (1,800) ft at the Allendale-Hampton County line at the southern edge of SRS (Aadland et al., 1995).

The Piedmont Province, which underlies the Coastal Plain Province, consists of Paleozoic metamorphic and igneous rocks overlain by Triassic-age lithified sediments. SRS does not generally use water from this province due to low yields and poor water quality (SGSD, 1997).

Hydrostratigraphic classifications of the SRS area are continually being revised, but a simple characterization is presented in Table (6.2). Aadland et al. (1995) argue convincingly for the importance of using a classification system that does not rely on a one-to-one relationship between hydrostratigraphic and lithostratigraphic units, because lithologic boundaries do not necessarily correspond to hydrogeologic boundaries. The resulting system differentiates between aquifer/confining systems, units, and zones.

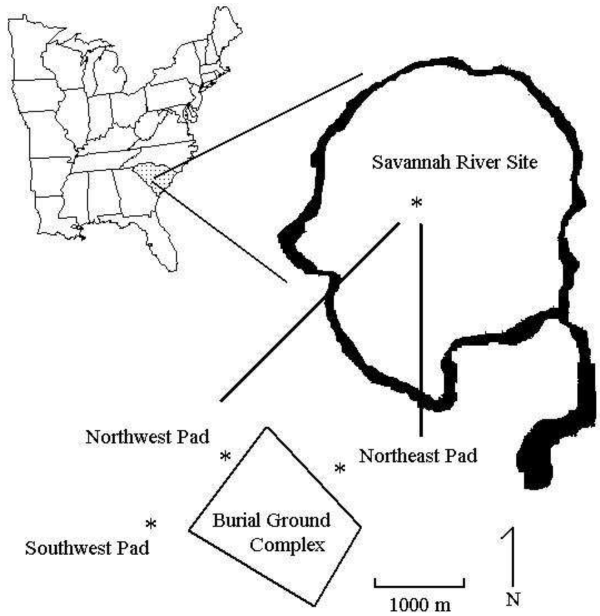

Figure 6.1: General location map for the Southwest Pad, Burial Ground Complex, Savannah River Site.

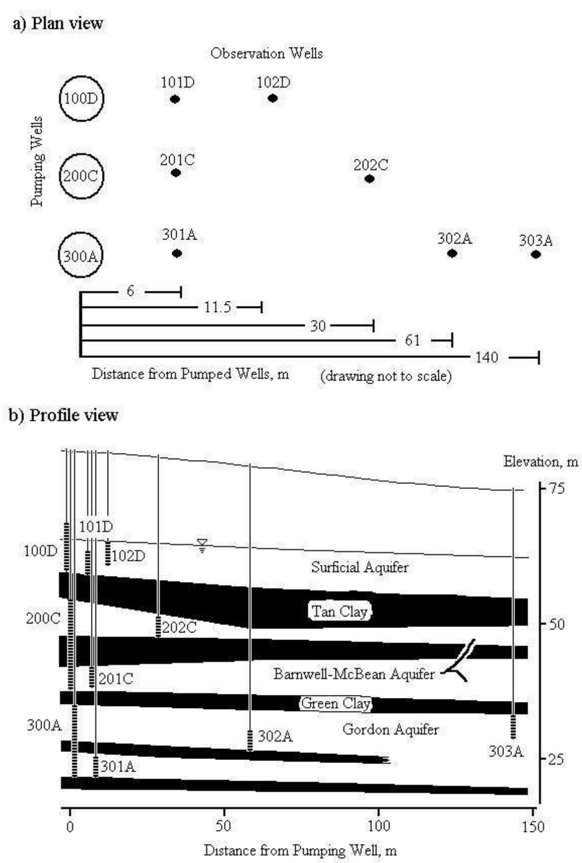

Figure 6.2: Site configuration for the Southwest Pad, a) plan view, b) profile view.

The uppermost aquifer unit in the Floridan Aquifer system is called the Upper Three Runs Aquifer. Groundwater flow beneath the site is south towards Four Mile Branch. The base of this unit occurs at the green clay, 30 to 50 m (100 to 160 ft) below the ground surface. The Surficial Aquifer is encountered at depths ranging up to 25 m (75 ft) below the ground surface near the F-Area seepage basins. Near the H-Area seepage basins, the water table is encountered at depths up to 15 m (50 ft). These depths correspond to 62.5 and 71.5 m (205 and 235 ft) above mean sea level (amsl) respectively. Saturated thicknesses range from 3 to 12 m (10 - 40 ft).

Rain recharges the aquifer and discharges occur at seeplines near Four Mile Creek and leakage occurs through the lower aquitard, called the Tan Clay. For the Surficial Aquifer, two tests were conducted. Hydraulic parameters were estimated using the steady-state Dupuit formula, yielding estimates of the hydraulic conductivity of 1.4×10^{-4} m/s (40 ft/day), and the transient Neuman curve fitting method, yielding a value of 1.9×10^{-4} m/s (55 ft/day). Kh from another test is estimated at to 0.8-1.1×10^{-4} m/s (24 to 32 ft/day) using the Dupuit method, and 0.7-0.8×10^{-4} m/s (20 - 24 ft/day) using the Neuman method. With both tests, Kh/Kv ratios were determined and indicated anisotropic conditions, which violates the Dupuit method conditions.

The Tan Clay aquitard is a semi-confining (leaky) layer formed by lenses of clay and sandy clay of varying thickness. Thicknesses range from 1.5 to 8 m (5 to 25 ft); the Tan Clay pinches out entirely in some areas. The zone exists between 50 to 58 m (165 and 190 ft) amsl beneath the F-Area Seepage Basins and between 56 to 66 m (185 to 215 feet) amsl beneath the H-Area Seepage Basins. The Tan Clay is encountered 6 to 34 m (20 to 110 ft) below the ground surface. Due to a head difference of 60 to 150 cm (two to five ft) between from the overlying surficial aquifer to the underlying Barnwell-McBean aquifer, flow is downward through this layer.

The Barnwell-McBean Aquifer is a poorly-defined, semi-confined aquifer ranging in thickness from 12 to 40 m (40 to 90 ft) from the F to the H-Area Seepage Basins. This layer is topographically influenced and depth to groundwater can be from 3 to 25 m (10 to 80 ft). Groundwater surfaces exist from 59 to 66 m (195 to 215 ft) amsl below the F-Area Basins and from 60 to 70 m (200 - 230 ft) amsl below the H-Area Basins. This zone consists mainly of sands and fine-grained material. Limestone can be found in the lower section of the aquifer. The Barnwell/McBean Aquifer was tested several times. Two full screen tests, and an *upper* and *lower* screen test were conducted resulting in steady-state K_h estimates of 2.8×10^{-5} and 1.4 $\times10^{-5}$ m/s (8 and 4 ft/day), respectively, using the steady state DeGlee method. Non steady-state estimates based on the transient Hantush-Jacob method were 2.8×10^{-5} to 3.2×10^{-5} m/s (8 to 9 ft/day) and 1.4×10^{-5} and 2.1×10^{-5} m/s (4 to 6 ft/day).

The Green Clay aquitard includes abrupt facies changes

Table 6.2: Representative lithostratigraphy and hydrostratigraphy of the Savannah River Site.

<table>
<tr><th colspan="3">Lithostratigraphy</th><th colspan="2">Hydrostratigraphy</th></tr>
<tr><th>Age</th><th>Group</th><th>Formation</th><th>Unit</th><th>System</th></tr>
<tr><td>Miocene</td><td>Hawthorne</td><td>Altamaha (Upland)</td><td rowspan="3">Upper Unit (Surficial Aquifer)</td><td rowspan="10">Floridan Aquifer System</td></tr>
<tr><td rowspan="9">Eocene</td><td rowspan="5">Barnwell</td><td>Tobacco Road</td></tr>
<tr><td>†Irwinton Sand</td></tr>
<tr><td>†Twiggs Clay</td><td>Tan Clay Aquitard</td></tr>
<tr><td>†Griffins Landing</td><td rowspan="3">Lower Unit (Barnwell-McBean Aquifer)</td></tr>
<tr><td>Clinchfield</td></tr>
<tr><td rowspan="3">Orangeburg</td><td>Tinker/Santee</td></tr>
<tr><td>Warley Hill</td><td>Green Clay Aquitard</td></tr>
<tr><td>Congaree</td><td rowspan="2">Gordon Aquifer</td></tr>
<tr><td rowspan="3">Black Mingo</td><td>Fishburne/Fourmile</td></tr>
<tr><td rowspan="2">Paleocene</td><td>Snapp/Williamsburg</td><td rowspan="2">Crouch Branch Aquitard</td><td rowspan="2">Meyers Branch Confining System</td></tr>
<tr><td>Ellenton</td></tr>
<tr><td rowspan="2">Cretaceous</td><td rowspan="2">Lumbee</td><td>Steel Creek / Peedee</td><td rowspan="2">Crouch Branch Aquifer</td><td rowspan="2">Dublin-Midville Aquifer System</td></tr>
<tr><td>Black Creek</td></tr>
</table>

†Members of Dry Branch Formation

from clay to silty and sandy material. Thicknesses range from 60 cm to 3 m (two to ten ft). This unit is located 32 to 38 m (105 to 125 ft) amsl. The permeability of this unit varies greatly; zones within this unit can be locally confining, semi-confining (leaky), or transmissive. Groundwater modeling efforts have established a vertical hydraulic conductivity of 6.4×10^{-10} m/s (1.8×10^{-4} ft/day). Laboratory samples have exhibited higher and lower values.

The Gordon Aquifer is the deepest aquifer in the Floridan Aquifer System and can be found at an elevation of approximately 300 m (100 ft) amsl. Thicknesses range from 20 to 30 m (70 to 100 ft). Pumping tests indicate a transmissivity of 2.5×10^{-3} m^2/s (2,300 ft^2/day), a hydraulic conductivity of 1.6×10^{-4} m/s (45 ft/day), and a storativity of 2.5×10^{-4}. This prolific aquifer can produce as much as a thousand liters per minute (several hundred gallons per minute).