Chapter 4

Surface Water Hydrology

The hydrologic response to precipitation is one of nature's wonders. In forested watersheds the runoff is gentle - delayed by thick layers of litter and roots that hold the soil and water in place. In urban watersheds, the water rushes unchecked into pipes that convey it rapidly to streams, carrying with it the litter and feces of countless people and pets.

4.1 Measuring Streamflow

As long as water is abundant, we seldom notice how much we use. Early water users pumped or diverted water to meet their needs, and were not particularly concerned about waste.

But as water becomes scarce, we tend to pay greater attention to our uses - how much and for what purposes. We also want to make sure that no one person or group gets more than their fair share. Thus is born the need to measure water.

Water flow in a channel, Q, is calculated using:

$$Q = \bar{v}\, A \tag{4.1}$$

where $\bar{v}$ is the average stream velocity and A is the cross-sectional area perpendicular to flow.

Stream discharge can be directly measured using field measurements of water velocity within the channel. Because the water velocity can be highly variable within a channel, multiple measurements of water velocity are needed at different points in the channel.

The stream discharge is calculated using the sum of discharges in specific sections within the channel:

$$Q = \sum_{i=1}^{n} Q_i \tag{4.2}$$

where Q_i is the flow in each section.

A detailed channel cross-section is often combined with a detailed velocity profile to give a more accurate estimate:

$$Q_i = v_i\, A_i = v_i\, W_i\, D_i \tag{4.3}$$

where A_i is the flow in an individual stream segment, and v_i, W_i, and D_i are the velocities, widths, and depths in each segment, respectively.

The velocity is not constant with depth in a channel, with the maximum velocity occurring at or near the surface, and a zero or very low velocity along bottom and sides. As a rule of thumb, the average velocity is commonly assumed to be at a depth of about 60% of the distance from the surface to the bottom of the channel. For deep rivers, multiple velocity measures taken at different depths can be used to provide a better average.

Rating Curve. Stream discharge is also not constant in time. Increasing discharges often result in increasing water levels, or *stages* within the channel. To account for temporal variations in discharge, channel measurements should be obtained at different stages. The relationship between stream stage and stream discharge is called the *rating curve.*

A rating curve is used to relate stream stage, h, to stream discharge, Q. The stream stage is the height of water, usually measured using a staff gage, which is just a vertical scale permanently attached to a point in the water so that the water level can be easily determined. The stage-discharge relationship commonly takes the form:

$$Q = a(h - h_o)^b \tag{4.4}$$

where h_o is a reference elevation corresponding to when $Q \to 0$. The staff gage should be placed in a pool upstream of a nickpoint (riffles, or falls) to assure that conditions are always subcritical, i.e., that the velocity head can be neglected.

Manning's equation. Manning's equation is commonly used to predict mean stream velocity, $\bar{v}$:

$$\bar{v} = \frac{c}{n} R^{2/3} S^{1/2} \tag{4.5}$$

where $c = 1$ in metric units and $c = 1.49$ in English units, n is Manning's coefficient (values provided in Table 4.1), $R = A/P$ is the hydraulic radius, with A being the stream cross-sectional area and P being the wetted perimeter where the stream meets the bed, and $S = \Delta h/\Delta x$ is the stream slope, with Δh being the change in total head and Δx being the downstream distance.

Table 4.1: Examples of Manning's n for various channel conditions.

Channel Condition			n
Channels	Straight	smooth hard bottom, uniform cross-section, no vegetation	0.020
		sandy/gravel bottom, variable cross-section, scattered vegetation	0.025
	Mostly Straight	irregular cross section, scattered rocks, increasing plants	0.030
	Winding	occasional pool and shoals, considerable rock or vegetation	0.040
		v. rocky, irregular cross-section, many pools and shoals and/or v. reedy	0.050
	Pools	v. reedy, sluggish, heavy channel vegetation	0.100
Floodplains	Bare soil, short grasses, scattered brush or timber		0.035
	Light brush and trees		0.060
	Heavy stand of timber with some down trees, some undergrowth		0.100

The stream slope is commonly taken as the average over a long reach, i.e., using contour intervals on a topographic map.

The hydraulic radius is approximately equal to the water depth, $R \approx D$, in a wide, rectangular channel because $A = W \cdot D$ and $P = W + 2D \approx W$ with $D \ll W$, so that we can simplify this to:

$$\bar{v} = \frac{c}{n} D^{2/3} \sqrt{S} \tag{4.6}$$

Control Structures. The most accurate way to measure water is to build a structure that allows us to precisely determine the flow. There are two general categories of control structures, *weirs* and *flumes*. While not intended for this purpose, even *culverts* can be used to measure streamflow, although not as accurately as a weir or flume. Table 4.2 summarizes these structures, and indicates there general strengths and weaknesses.

Table 4.2: Types of channel control structures.

Weirs:

- stilling basin is located upstream of weir
- water level recorder is used to measure stage in stilling basin
- outlet structures include rectangular, triangular (v-notch), and Cipolletti (trapezoidal) shapes
- weir crests can be broad (flat lip) and sharp (knife-blade) crested
- flow is subcritical upstream of crest, supercritical downstream
- weirs collect sediments in the stilling basin, debris on weir crest

Flumes:

- no stilling basin, only a narrow throat
- regular approach section
- passes sediment easily
- woody debris can be a problem

Culverts:

- Four combinations of flow equations, flooded vs. open upstream, flooded vs. open downstream
- Culvert should be a regular shape, round or rectangular, with no debris

Weirs. Weirs are structures built into the stream channel to provide a better estimate of stream discharge. There are two general types of weirs; *sharp-crested*, which have a vertical, knive-edge weir blade that water flows over; and *broad-crested*, which have a broad lip or surface that water flows over.

Weirs may not provide accurate estimates in several situations. One source of error occurs when the weir blade becomes blocked by ice or floating debris, such as leaves and branches. Another source of error arises when the weir pool fills with sediments, resulting in an inaccurate estimate of the total head.

Sharp-Crested Weirs. A sharp-crested weir is constructed so that the flowing water passes over a vertical, knife-edge, thus minimizing resistance with the weir blade. Various formula related to measuring flow over a sharp-crested weir are in Table 5.13.

A dam placed in the river or stream with a hole below the upstream water level can be used to determine the flow. In this case, a flooded, circular orifice equation is used:

$$Q = C_d\ A\ \sqrt{2gh} \tag{4.7}$$

where Q is the flow rate in cubic feet per second, C_d is a dimensionless discharge coefficient, with a range of $0.5 < C_d < 1$ and $C_d = 0.6$ commonly used, $A = \pi r^2$ is the orifice cross-sectional area, r is the outlet radius, $g = 9.807\ m/s^2$ is the gravitational constant, and h is the height of water above the center of the orifice.

For flow over the top of a weir, the corresponding equation is:

$$Q = C\ A\ h^{1/2} \tag{4.8}$$

where $C = C_d\sqrt{2g}$ and A is the cross-sectional area perpendicular to flow. For a rectangular opening, the area is $A = Wh$, so that:

$$Q = C\ W\ h^{3/2} \tag{4.9}$$

Table 4.3: Sharp-crested weir equations.

Type	Formula†
Flooded Orifice	$Q = C\ A\ h^{0.5}$
Rectangular	$Q = C\ W\ h^{1.5}$
Triangular	$Q = C\ \tan\ \frac{\alpha}{2}\ h^{2.5}$

† Neglects contraction effects along weir blade edges. h is elevation of water surface in stilling basin, A is area of opening, W is width of weir, α is weir angle.

and for a triangular (V-notch) opening:

$$Q = C\ \tan\frac{\alpha}{2}\ h^{5/2} \tag{4.10}$$

because $A = h^2 \tan\frac{\alpha}{2}$.

Broad-Crested Weirs. A broad-crested weir consists of an outflow structure over which water flows for some distance before falling over the downstream edge.

The total head equation can be written as:

$$h = D + \frac{v^2}{2g} \tag{4.11}$$

where h is the total head, D is the stream depth, v is the stream velocity, and g is the gravitational constant, and where the pressure head is neglected. Conservation of mass for conditions of steady flow requires that:

$$Q = WDv \qquad or \qquad v = \frac{Q}{WD} \tag{4.12}$$

where Q is stream discharge and W is the stream width. Combining equations yields:

$$h = D + \frac{1}{2g}\left(\frac{Q}{WD}\right)^2 \tag{4.13}$$

The minimum energy occurs when the derivative of total head with respect to depth is zero:

$$\frac{dh}{dD} = 1 - \frac{1}{gD}\left(\frac{Q}{WD}\right)^2 = 0 \tag{4.14}$$

or:

$$D = \sqrt[3]{\frac{Q^2}{gW^2}} \tag{4.15}$$

Substitution yields:

$$v = \sqrt[3]{\frac{gQ}{W}} \tag{4.16}$$

and

$$h = \sqrt[3]{\frac{Q^2}{gW^2}} + \frac{1}{2}\sqrt[3]{\frac{Q^2}{gW^2}} \tag{4.17}$$

Inspection of this equation reveals that the velocity head is just one-half the elevation head:

$$\frac{v^2}{2g} = \frac{D}{2} \tag{4.18}$$

so that:

$$D = 2h/3 \qquad v = \sqrt{2gh/3} \tag{4.19}$$

For a rectangular, broad crested weir, this becomes

$$Q = W\sqrt{g(2h/3)^3} \tag{4.20}$$

or

$$Q = C\,Wh^{1.5} \tag{4.21}$$

where $C = \sqrt{8g/27}$.

Flumes. Flumes provide an alternative method for estimating discharge. Flumes require no upstream stilling pond, and allow sediment to pass unimpaired through the structure. Ice, leaves and other debris can still affect readings, however.

The *H-type* flumes developed by the U.S. Department of Agriculture are useful for measuring discharge in sediment-laden streams. A small drop is required downstream of H-type flumes, which can be difficult to achieve in level channels.

Another popular flume design is the *Parshall* flume, which uses a constriction in the width and depth of the channel to force the flow to become supercritical downstream. No drop is needed downstream of the structure, allowing its use in level channels.

Culverts. Rather than having to construct a control structure, we often find culverts already in the channel that were built to route water under roads. Culverts are often round or rectangular, providing a regular section for measuring discharge.

There are four combinations of two general conditions, upstream flooded or open, downstream flooded or open. Perhaps the simplest condition occurs when both the upstream and downstream conditions are flooded. These are fully submerged conditions that are consistent with flow through a pipe. In this case, the velocity is estimated using the difference in elevation between the upstream and downstream water levels, along with the culvert length and diameter.

Another condition that can be analyzed occurs when subcritical (i.e., flooded) conditions are present upstream, and supercritical (i.e., open) conditions are present at the downstream end of the culvert. A broad-crested weir equation can be used in this case.

When the upstream condition is open, then a channel flow conditions exist, which can be determined using Manning's equation. Necessary information includes the culvert roughness, hydraulic radius, and slope.

Problems

1. What does a rating curve of $Q = ah^{1.49}$ tell you about the channel shape?

2. How would changing the Manning's coefficient affect onsite vs offsite (downstream) flooding?

4.2 Stormwater Hydrographs

Let us first examine the physical flow of water, putting the chemical and biological aspects aside until the next chapter. We can look at what causes stormwater, and then examine how land uses alter stormwater behavior. We shall see how the alteration of landscapes from forest to agriculture to urban uses increases the magnitude and timing of flood peaks and volumes.

Stormwater is the rapid response of a stream to a precipitation event. Unlike baseflow, which responds slowly to atmospheric inputs, stormwater responds immediately - both rising and falling quickly with the weather. Evidence of stormwater in an urban environment is easy to obtain - puddles, flooded streets, and raging storm sewers. Evidence in agricultural settings is also readily seen - sheet flow (i.e., overland flow), flooded fields, and rille erosion.

Overland flow in urban and agricultural environments results from low infiltration rates due to pavement and soil compaction. This flow type of overland flow is called *surface runoff* or *Hortonian flow*. Finding stormwater sources in forested areas is more difficult, however, because of the higher permeability of forest soils. Water readily infiltrates into worm and animal burrows, and into decayed roots channels. Even though the water is underground, it can still move quickly through macropores to nearby streams. This flow, called *subsurface runoff*.

Yet overland flow can still occur in forested environments when sufficient rainfall causes saturation of the underlying soil. In this case, it is not a reduction in soil permeability that inhibits infiltration, but rather the reduction of the hydraulic gradient into the soil. Subsurface saturation essentially raises the water table to the ground surface, causing near-hydrostatic conditions in the subsurface.

Stormwater Volume and Depth

The total volume of stormwater is often a source of concern, especially for reservoir and detention basins that are designed to contain a specific volume of water. In this

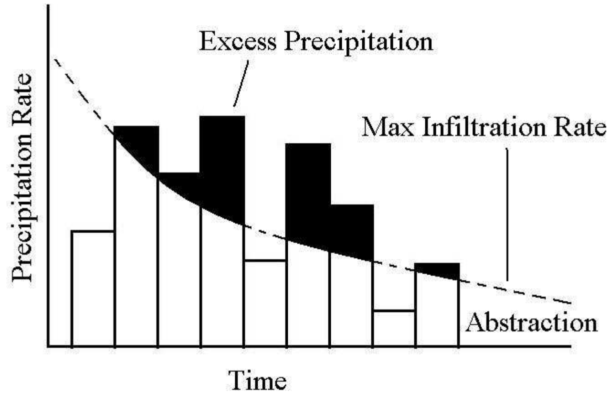


Figure 4.1: Excess precipitation resulting from limited soil infiltration capacity.

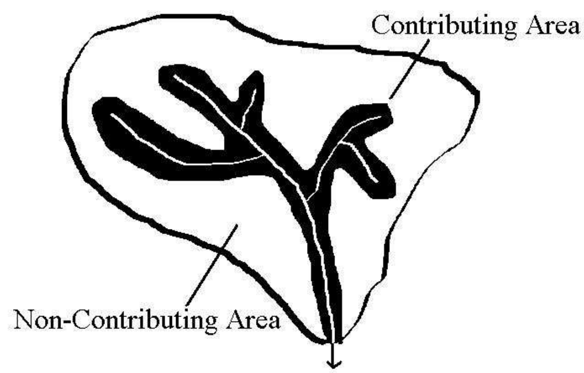


Figure 4.2: Excess precipitation resulting from limited soil moisture storage.

case, the rate of runoff is not of concern, and only the cumulative inflows are an issue:

$$Q = \sum_i \frac{Q_i}{A} = \sum_i q_i \tag{4.22}$$

where Q is the total runoff depth, Q_i is the runoff depth at time i, and q_i is the runoff rate per unit area, A. Note that runoff can also be written as a *volume*, $V = Q\ A$, by multiplying the *runoff depth*, Q, by the *watershed area*, A.

Two fundamentally different models of stormwater generation have been proposed. The first posits that runoff occurs when the rainfall intensity exceeds the soil infiltration rate. In this model, high intensity rains are more likely to cause stormflow than low intensity storms.

Alternatively, a second approach posits that soil storage limits the *volume* of water that the soil can absorb. In this model, storms with more rainfall *depth*, as opposed to greater *intensities*, result in greater runoff. Areas that are saturated contribute to stormflow, while areas that are unsaturated absorb all the rainfall and do not contribute to stormflow.

To illustrate this concept, let us define the total runoff efficiency, $R = Q/P$, as the ratio of the total depth of runoff, Q, to the total precipitation depth, P. The usual limits of the total runoff efficiency $0 \leq R \leq 1$ reflect the fact that stormwater runoff does not exceed rainfall, and can not be negative. During a storm, the runoff efficiency is commonly observed to increase over time as the soil becomes saturated.

At any given time, we can find the precipitation rate using $p = dP/dt = \Delta P/\Delta t$ which is just the change in precipitation depth per unit time. Likewise, the runoff rate is $q = dQ/dt = \Delta Q/\Delta t$. The runoff rate, q, can be calculated using:

$$q = \frac{0 \cdot A_1 + p \cdot A_2}{A_1 + A_2} = p \cdot a_2 \tag{4.23}$$

where A_1 is the area where the rainfall rate is less than the infiltration rate, A_2 is the area of impervious surfaces, and $a_2 = A_2/(A_1 + A_2)$ is the proportion of area that is impervious.

This equation implies that no runoff is generated in areas where the rainfall is less than the infiltration rate, because all of the rainfall is absorbed into the soil. It also implies that, once the soil is saturated, all of the rainfall is converted into runoff. This is the essence of the *contributing area* concept, which posits that only those saturated parts of the landscape that are interconnected with streams actually contribute to stormflow.

We can pursue this concept by defining an *abstraction*, or infiltration, rate, f, as:

$$f = p - q = p\ (1 - a_2) = p \cdot a_1 \tag{4.24}$$

where a is the proportion of pervious surfaces. This means that all infiltration occurs only in those areas which are not saturated. While some residual infiltration may occur in saturated areas, many saturated areas can actually be *exfiltrating*, or discharging water from the subsurface to the surface, so that the net effect of infiltration plus exfiltration may balance each other.

Curve Number Method

We can use the *Curve Number* method to further explore this concept. The Curve Number method is a widely used approach for estimating the runoff from different landscapes. The fundamental relationship between storm effective rainfall depth, P and runoff depth, Q, embedded within the method is:

$$\frac{Q}{P} = \frac{F}{S} = \Theta \tag{4.25}$$

where Θ is the relative saturation of the soil, which varies between $0 \leq \Theta \leq 1$, $F = P - Q$ is the abstraction depth, and S is the maximum soil storage depth.

Replacing the abstraction depth and solving for the runoff depth results in:

$$Q = \frac{P^2}{P + S} \tag{4.26}$$

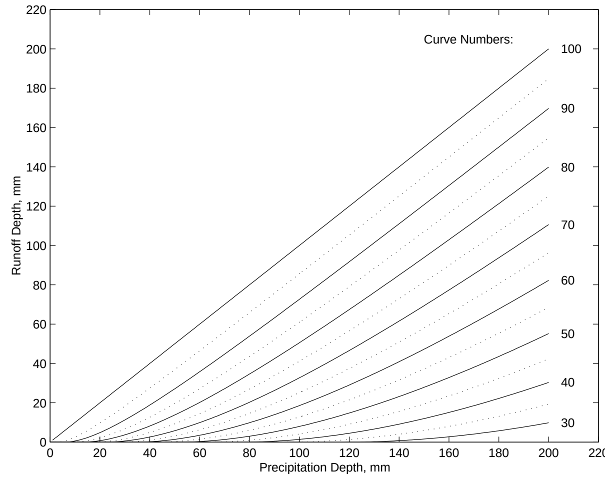


Figure 4.3: Curve Number method for estimating excess precipitation.

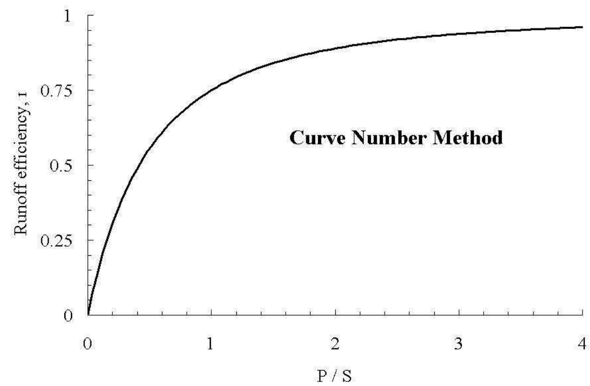


Figure 4.4: Contributing Area Runoff Efficiency using the Curve Number Method.

One can calculate the runoff efficiency from this function as:

$$r = \frac{q}{p} = \frac{dQ}{dP} = \frac{2P}{P+S} - \frac{P^2}{(P+S)^2}$$
$$= 1 - \frac{1}{(1 + P/S)^2} = 1 - (1 - \ \theta)^2 \tag{4.27}$$

This function is plotted in the accompanying figure. Note that the runoff efficiency increases as the water content increases - which is consistent with the notion that the contributing area for runoff increases with time.

To use the curve number method, we first find the effective precipitation, $P = P_o - I_a$, P_o is the observed precipitation and I_a is the initial abstraction of water required for runoff to begin. Combining both Equations yields:

$$Q = \frac{P^2}{P+S} = \frac{(P_o - I_a)^2}{(P_o - I_a) + S} = \frac{(P_o - 0.2S)^2}{P_o + 0.8S} \tag{4.28}$$

where $I_a = 0.2\ S$ is assumed. We then use the curve number, CN, to find the maximum storage capacity of

Table 4.4: Curve Numbers as a function of land use and soil conditions.

Land Use	Hydrologic Soil Group			
	A	B	C	D
Woods and Forests:				
Good (no grazing; brush covers ground)	30	55	70	77
Fair (grazing but not burned; some brush)	36	60	73	79
Poor (small trees/brush destroyed by over-grazing or burning)	45	66	77	83
Open Spaces (lawns, parks, golf courses, cemeteries, etc.):				
Good (grass covers $> 75\%$ of area)	39	61	74	80
Fair (grass covers $50 - 75\%$ of area)	49	69	79	84
Pasture or Range Land:				
Brush (good, $> 75\%$ ground cover)	30	48	65	73
Meadow (grass, no grazing, mowed for hay)	30	58	71	78
Good ($50 - 75\%$ ground cover; not heavily grazed)	39	61	74	80
Poor ($< 50\%$ ground cover or heavily grazed)	68	79	86	89
Cultivated (Agricultural Crop) Land:				
With conservation treatment (terraces, contours)	62	71	78	81
Without conservation treatment (no terraces)	72	81	88	91
Residential Areas:				
1 Acre lots, about 20% impervious	51	68	79	84
1/2 Acre lots, about 25% impervious	54	70	80	85
1/4 Acre lots, about 38% impervious	61	75	83	87
1/8 Acre lots, about 65% impervious	77	85	90	92
Industrial Districts (72% impervious)	81	88	91	93
Commercial and Business Districts (85% impervious)	89	92	94	95
Streets and Roads:				
Dirt	72	82	87	89
Gravel	76	85	89	91
Paved with curbs and storm sewers	98	98	98	98
Paved parking lots, roofs, driveways	98	98	98	98

The hydrologic soil groups used in the table are:

Soil Group A High infiltration (low runoff); Sand, loamy sand, or sandy loam; Infiltration rate greater than 1 cm/hr when wet.

Soil Group B Moderate infiltration (moderate runoff); Silt loam or loam; Infiltration rate 0.5 to 1 cm/hr when wet.

Soil Group C Low infiltration (moderate to high runoff); Sandy clay loam; Infiltration rate 0.1 to 0.5 cm/hr when wet.

Soil Group D Very low infiltration (high runoff); Clay loam, silty clay loam, sandy clay, silty clay, or clay; Infiltration rate less than 0.1 cm/hr when wet.

the soil, S, in units of *inches*:

$$S = \frac{100}{CN} - 10 \qquad or \qquad CN = \frac{100}{S+10} \tag{4.29}$$

Curve Numbers are obtained using the type of land use, vegetation, and soil factors as variables.

Antecedent moisture conditions are also important, in that they increase the amount of water stored in the soil, thus increasing the water content and reducing the amount of storage available in the soil. Three types of moisture conditions are generally used:

Dry Soils are dry, but not to the wilting point - decrease the CN

Normal Typical, or average, conditions - use the standard CN

Wet Recent heavy rainfall with saturated soils - increase the CN

Other functions can also be used to represent the relationship between runoff and soil moisture storage. One approach would be to assume that soils are spatially variable across the landscape, and then assign a unique distribution of moisture storages to the soil.

Estimating Peak Discharge

Peak discharge, Q_p, is another widely used measure of stormwater impact because the maximum damage is commonly associated with the peak magnitude. Roads are washed away - and worse yet, lives are lost as cars and homes are carried away by the swirling floodwaters.

While runoff is generated at various points across the landscape, stormwater discharge is only measured at a single point downstream. Thus, the rate at which stormwater is generated across the landscape must be converted into a point discharge. We do this by incorporating the runoff travel time between the source and the measurement location.

There are several methods for estimating peak discharges from small watersheds - i.e., from catchments less than approximately 40 ha.

Rational Method. The rational method is frequently used to related land use and precipitation intensity to the peak runoff rate: The

$$Q_p = CIA \tag{4.30}$$

where C is a land use factor, I is the rainfall intensity for duration equal to time of concentration, and A is the watershed area.

The time of concentration, t_c, is the time of travel for stormwater runoff from the most distant point in the watershed to reach the outlet. If the watershed is $L = 600m$ in length, and the stormwater velocity is $v = 1\ m/s$, then

Table 4.5: Rational Method Runoff Coefficient, C

Ground Cover	C	
	Low	High
Lawns	0.05	0.35
Forest	0.05	0.25
Cultivated land	0.08	0.41
Meadow	0.10	0.50
Parks, cemeteries	0.10	0.25
Unimproved areas	0.10	0.30
Pasture	0.12	0.62
Residential areas	0.30	0.75
Business areas	0.50	0.95
Industrial areas	0.50	0.90
Streets		
bricks	0.70	0.85
asphalt	0.70	0.95
concrete	0.70	0.95
Roofs	0.75	0.95

the time of concentration is $t_c = L/v = 600\ s$ or $10\ min$. The time of concentration is used for selecting the appropriate precipitation event because the largest runoff peaks are associated with storm events of this duration.

Note that the peak discharge is linear with the watershed area, so that we can write: The

$$q_p = \frac{Q_p}{A} = C\ I \tag{4.31}$$

where q_p is the runoff per area, which is a water *yield* - where a yield is an amount of some material (water, pollution, etc.) per unit area.

Channel Geometry. Another simple model of peak runoff uses the channel width, w:

$$Q_p = a\ w^b \tag{4.32}$$

This approach assumes that the channel width is in equilibrium with peak discharges, so that larger peak flows are reflected in larger channel widths.

The channel geometry approach is not appropriate when the characteristics of the contributing area change over time. This is because the channel may not immediately respond to upstream alterations. As storm peaks increase - due to development, for example - then the channel will slowly widen until a new equilibrium is achieved. Likewise, if a degraded watershed is restored using riparian buffers and stormwater retention basins, then the channel will slowly establish a new equilibrium shape.

Watershed Characteristics. Note that the rational method does not directly account for variations in topography or hydrographic features, such as lakes and ponds. More elaborate empirical models can be used, such as the

Benson model:

$$Q_p = a\ A^b\ S^c\ S_t^d\ I_b^e\ L^f \tag{4.33}$$

where A is the watershed area, S is the main channel slope, S_t is the area of lakes and ponds, I_b is the maximum 24-hour, 10-year precipitation depth, and L is the main channel length.

Another model, proposed by *Scott*, is:

$$Q_p = a\ A^b\ E_m^c\ S_t^d\ P_s^e\ I^f\ T^g \tag{4.34}$$

where E_m is the mean altitude, P_s is the average May to September precipitation depth, I is the maximum 24-hour, 2-year precipitation depth, and T is the mean January temperature.

And finally, the *Borland* model is:

$$Q_p = a\ A^b\ E_m^c\ S_h^d\ S^e\ S_t^f\ P_a^g\ P_s^h\ I^j\ L_a^j\ L_o^k \tag{4.35}$$

where S_h is the watershed shape, P_a is the average October to April precipitation depth, L_a is the latitude, and L_o is the longitude.

These models are formed by using peak stormflows at gages where watershed information is available. Normally, these models are regional - in that they can only be applied in the area where they were developed.

To form you own model, logarithmic transforms are used to linearize the equations:

$$y = y_o + b\ x_1 + c\ x_2 + d\ x_3 + \cdots \tag{4.36}$$

where $y = \log Q_p$, $y_o = \log a$, $x_1 = \log(A)$, etc. Note that any number of factors can be added, thus improving the *fitting* or *calibration* error. *Multicollinearity* is major problem with this approach, however, causing large *prediction* errors. One must always check the prediction uncertainties to make sure that each added parameter further decreases the prediction error.

Unit Hydrographs

Unit hydrographs are used when the shape of a stormwater hydrograph is desired. It is used to predict the stormflow hydrograph for conditions where one unit of effective precipitation (net runoff) falls on a watershed during one time period. The duration, time to peak, and peak discharge are all represented using a unit hydrograph.

A common shape to use is the *triangular* unit hydrograph - where the duration of the hydrograph, t_c, is the time of concentration within the watershed, and is the base of the triangle. The peak discharge, Q_p, is the height of the triangle, and the maximum height occurs at the time to peak, t_p.

The area of the triangle, $Q = t_c\ Q_p/2$, represents the total volume of stormwater runoff. We normally set the total volume equal to one unit of runoff, hence the name *unit* hydrograph. Note that reducing the time of concentration requires a higher peak in order to maintain a unit runoff.

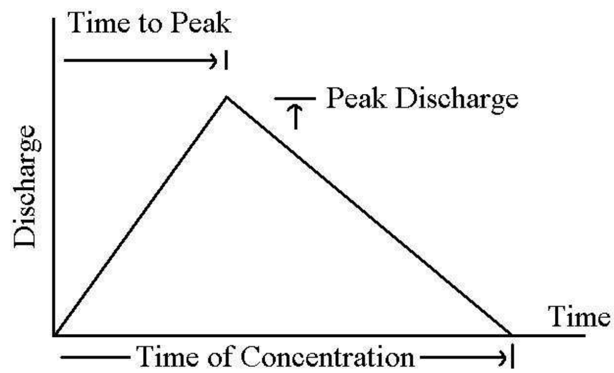


Figure 4.5: Triangular Unit Hydrograph.

Other shapes besides a triangle can certainly be used - the only constraint is that the area under the curve must equal zero. The basic concept that a unit of runoff has a specific shape that is determined by the watershed.

Linear Systems. A linear system is composed of inputs, outputs, state variables, and parameters. In this case, the input is the observed precipitation, the output is the predicted hydrograph, the state variables are watershed characteristics such as antecedent moisture conditions, and the parameters are the properties of the unit hydrograph.

Once a unit hydrograph has been constructed, we then use linear theory to manipulate the unit hydrograph for various precipitation events:

- Linear theory assumes that responses are *linear* - a doubling of the net precipitation causes a doubling of the predicted stormwater discharge.
- We also assume that the system is *time-invariant* - the response to precipitation in one hour is the same as the response in any hour.
- We must also assume that the responses are *independent* of each other - the runoff in one hour does not affect the runoff in another hour.

For example, for a linear system, a unit input results in a unit response

- If $y = f(x)$, then $cy = cf(x) = f(cx)$
- $y = mx + b$ is not a linear system because $2\ (mx + b) \neq m\ (2x) + b$.

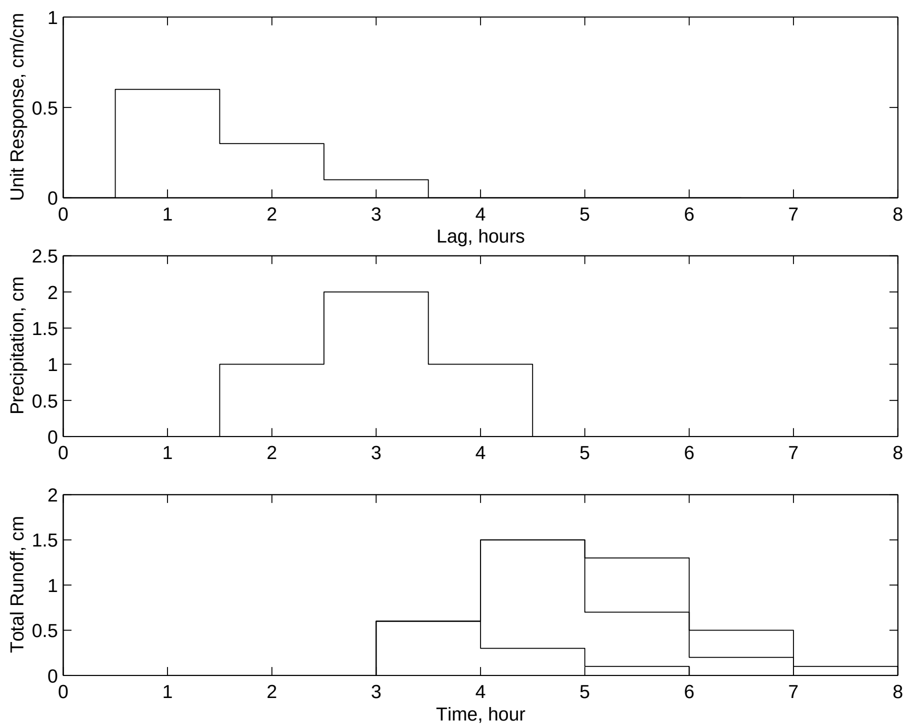
If the system is linear, then an input that is different than unity means that the output can be simply scaled by the input. Thus, if we had two units of net precipitation, then we would double the volume of the unit hydrograph by doubling the discharge heights while holding the time base constant.

Table 4.6: Example of convolution using a unit hydrograph.

Hour:	2	3	4	Sum
Net Precipitation, $x(t)$	1	2	1	4

Delay:	0	1	2	3	4	Sum
Unit Hydrograph, $h(i)$	0	0.6	0.3	0.1	0	1.0

Hour:	2	3	4	5	6	7	8	Sum
Response to hour 10	0	0.6	0.3	0.1	0	-	-	1
Response to hour 11	-	0	1.2	0.6	0.2	0	-	2
Response to hour 12	-	-	0	0.6	0.3	0.1	0	1
Discharge, $y(t)$	0	0.6	1.5	1.3	0.5	0.1	0	4



Convolution. Convolution is used to scale and lag the unit hydrograph to reflect changing rainfall intensities over time. For example, two units of rainfall at the same time cause twice the runoff, but no change in lag, while two units of rainfall at different times, are the sum of individual rainfall events, where the two events are lagged by the delay between the two events.

Mathematically, the convolution operation is defined using the *star* operator:

$$y = h * x = \int_{-\infty}^{t} h(\tau)\ x(t-\tau)$$

$$= \sum_{i=0}^{m} h(i)\ x(t-i) \tag{4.37}$$

where y is the system response, h is the unit response function, $*$ is the convolution operator, and x is the input. The three steps involved are:

1. Multiply unit response function, $h(i)$, by input, $x(t)$, for time step t
2. Shift to next hour and repeat Step 1, starting response at beginning of input interval
3. When all inputs have been multiplied by the unit hydrograph, add all responses

Deconvolution: Finding the Unit Hydrograph. To find the unit hydrograph, we need to have two time series, one of rainfall and the second of streamflow. Ideally, we would use data that are similar to the ideal prediction conditions, also call the *design storm*.

This means that we should use data from flood periods if we wish to predict a unit hydrograph that is appropriate for flood conditions, and data from drought conditions if we wish to create a unit hydrograph appropriate for drought conditions. In other words, given that the unit hydrograph is probably dependent upon a state variable, it is best to use data sets for which the state variable is most similar to the prediction requirements.

To estimate the coefficients of the unit hydrograph, we construct a *regression deconvolution* equation of the form:

$$y(t) = h(0)\ x(t-0) + h(1)\ x(t-1) + h(2)\ x(t-2) + \cdots + h(n)\ x(t-n) \tag{4.38}$$

where n is the time *memory* of the system, equal to the time of concentration.

Various methods can be used to estimate the unit hydrograph without streamflow data, including Snyder's, SCS Methods, Gray's, Expey's, Clark's, Nash's, and the Colorado method.

Problems

1. Explain how reducing the soil moisture storage capacity from 10 cm to 1 cm affects the volume of stormwater generated.
2. Calculate the Curve Number for soil moisture storages of 0, 1, 3, 5, 10, 30 and 50 cm.
3. Explain why shortening the time of concentration increases the peak discharge for a unit hydrograph.
4. Explain how changing the time to peak for a unit hydrograph might affect the time of concentration and the peak discharge.
5. Using the precipitation and discharge data in Table 4.6, use regression deconvolution to estimate the unit hydrograph.

4.3 Flood Routing

Once a hydrograph has been generated for a site, we are commonly asked to predict what happens to the flood as it moves downstream. There are generally two issues:

- How big will the flood peak be?
- When will the flood peak reach us?

To answer these questions, we must *route* the flood from the upstream point of observation to some downstream point or points.

Flood Translation

Translation is the movement of a flood wave through a channel without alteration. The peak discharge remains unchanged, as does the shape of the flood wave. The only change is the delay caused by the time of travel between the two points.

Thus, if $Q(t, x_a)$ is the discharge over time at location x_a, then the observed discharge at some downstream location x_b is just:

$$Q(t, x_b) = Q(t-\tau, x_a) \tag{4.39}$$

where τ is the time of travel between x_a and x_b, which can be calculated using:

$$\tau = \frac{x_b - x_a}{c} \tag{4.40}$$

where c is the velocity of the flood wave, which is also called the *celerity*.

Flood wave velocity. The water velocity in a stream can be expressed as:

$$\bar{v} = \frac{Q}{A} \tag{4.41}$$

This would represent the movement of a tracer or boat in the water as it drifts downstream.

The velocity of wave is different, however. Much like a stone cast into a pond that generates ripples that spread quickly to the shore, the wave velocity is an energy velocity, calculated using:

$$c = \frac{dQ}{dA} \tag{4.42}$$

where c is the wave velocity, or *celerity*. This is the rate at which a flood wave or a change in streamflow propagates down the channel.

This means that a flood wave travels at a speed determined by the *incremental* change in discharge with respect to the channel area. We can calculate this incremental velocity by substituting the discharge, $Q = vA$:

$$c = \frac{d(vA)}{dA} = v + A\ \frac{dv}{dA} \tag{4.43}$$

Note that the wave velocity equals the water velocity plus a second term that depends upon whether the velocity increases or decreases as the channel gets larger.

For flows that are within the channel, the velocity usually increases with channel depth because the viscous drag on the streambed is reduced. This results in a *positive* second term, which means that the flood wave velocity is *faster* than the water velocity.

For flows that spill out over the floodplain, the velocity usually decreases because the viscous drag on the floodplain is increased. This results in a *negative* second term, which means that the flood wave velocity is *slower* than the water velocity.

The *kinematic ratio* describes the ratio of the wave velocity to the water velocity:

$$k = \frac{c}{v} \tag{4.44}$$

A celerity greater than one is generally observed, but can be less than one in special cases, as we will show below.

In practical terms, a boater on a river would see the flood wave catch up and pass them for $k > 1$, while they would catch up and pass a flood wave if $k < 1$. Thus, there are two factors that delay the arrival of a flood peak, τ:

1. The shape of channel changes the flood wave velocity relative to the water velocity, i.e., the kinematic ratio, and
2. The shape of the channel changes the water velocity by changing the resistance within channel

We can insert the celerity definition to yield:

$$k = \frac{c}{v} = 1 + \frac{A}{v}\ \frac{dv}{dA} \tag{4.45}$$

For cases where the velocity does not change with area, the second term disappears, so that the kinematic ratio goes to one. For cases where the velocity increases with increasing cross-sectional area, then the kinematic ratio is greater than one, and is less than one otherwise.

In a wide, rectangular channel, we can assume a constant width so that:

$$k = 1 + \frac{D}{v}\ \frac{dv}{dD} \tag{4.46}$$

The relationship between v and D is given by Manning's equation, so that:

$$\frac{dv}{dD} = \frac{1}{n}\sqrt{S}\ \frac{dD^{2/3}}{dD} = \frac{2}{3}\frac{v}{D} \tag{4.47}$$

so that:

$$k = \frac{5}{3} \tag{4.48}$$

This means that a flood wave will move downstream about 2/3 faster than an object traveling in the water.

As the stage rises above the floodplain levees, however, the wetted perimeter increases rapidly, thus slowing the average velocity. In this case, dv/dA is negative, meaning that the average water velocity decreases with increasing stage. This results in a kinematic ratio less than one.

Transmission Losses *Losing* streams are watercourses that have substantial amounts of water losses to underlying aquifers. Unlike *gaining* streams that increase in size as they flow toward the ocean, losing streams slowly decrease in size, and may even disappear before they reach the ocean.

These streams are normally found in desert environments, where the rainfall is much less than the evapotranspiration rate. Water from cooler, mountainous regions flows down into the desert, and is quickly infiltrated into the streambed, recharging underlying aquifers. Losing streams might also be found in wetter climates where substantial ground-water pumping depletes aquifers near streams.

A simple infiltration model developed by Leonard Lane is:

$$Q(t, x_b) = Q(t - \tau, x_a)\ \exp(-k\ (x_b - x_a)) \tag{4.49}$$

where $Q(t, x_b)$ is the downstream discharge, $Q(t, x_a)$ is the upstream discharge, $x_b - x_a$ is the distance between the two locations, and k is an infiltration constant.

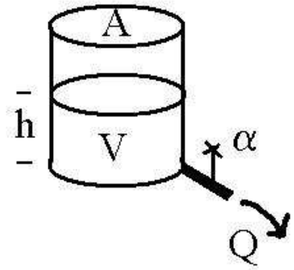


Figure 4.6: Linear Reservoir Definition Sketch.

Linear Reservoir Routing

The linear reservoir model is another simple model of water flow in channels. Consider a simple tank reservoir with a volume of water, $V = h\ A$, equal to the depth, h, and the area of the tank, A. Let us suppose that there is a valve that releases water from the bottom of the tank, and the outflow from the tank $Q = \alpha\ h$, is a function of how far open the valve is, α, and how high the water level is in the tank, h. As the water level drops, the outflow from the tank should decrease.

We can also say that the outflow must be due to a release of water from storage within the tank, or:

$$Q = -\frac{dV}{dt} = -A\ \frac{dh}{dt} \tag{4.50}$$

which just means that the outflow must equal the time rate of change of the volume of water in the tank, which is related to the rate of change in water level in the tank.

Combining both equations yields:

$$Q = \alpha\ h = -A\ \frac{dh}{dt} \tag{4.51}$$

Rearranging terms yields:

$$\frac{\alpha}{A}\ dt = -\frac{dh}{h} \tag{4.52}$$

Integration yields:

$$\frac{\alpha\ t}{A} + C = -\ln h \tag{4.53}$$

where C is the constant of integration. This can be simplified to:

$$h = h_o\ \exp(-\beta t) \tag{4.54}$$

where $h_o = h(t = 0)$ and $\beta = \alpha/A$. Substituting into the equation above yields:

$$Q = \alpha\ h = Q_o\ \exp(-\beta t) \tag{4.55}$$

where $Q_o = Q(t = 0)$. These are *exponential decay* equations, which means the values diminish at a slowing rate over time. Such a curve is typical of *first-order* streams, i.e., streams that are the smallest permanent streams in a landscape.

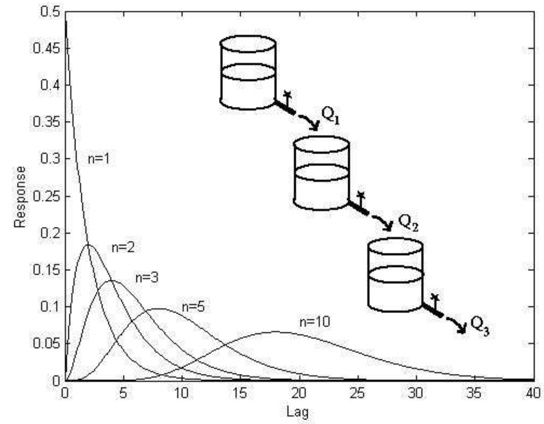


Figure 4.7: Cascade of linear reservoirs.

We can imagine a second order stream as being a second reservoir located below the first reservoir. In this case, the outflow from the first reservoir is the inflow to the second, and the outflow from the second is the inflow to the third. This series of reservoirs is called a *cascade* of linear reservoirs, or *Nash* model. The equation that describes these can be readily found using the Laplace transform method, giving a gamma function that looks just like many flood waves:

$$h(\tau) = \frac{\tau^{n-1}\ \beta^n\ \exp(-\beta\tau)}{(n-1)!} \tag{4.56}$$

where n is the number of reservoirs in series, or, equivalently, the expected response in an n^{th}-order stream.

For a number of linear reservoirs in *parallel*, we have:

$$\begin{aligned} Q = {} & Q_1 \exp(-\beta_1 t) + Q_2 \exp(-\beta_2 t) \\ & + Q_3 \exp(-\beta_3 t) + \cdots + Q_n \exp(-\beta_n t) \end{aligned} \tag{4.57}$$

Constant Inputs. We can also consider the case when a constant input, I, such as upstream inflows, or precipitation, are present

$$\frac{dV}{dt} = A\ \dot{h} - I \tag{4.58}$$

where $\dot{h} = dh/dt$, so that

$$A\ \dot{h} + k\ h = I \tag{4.59}$$

A common technique for solving ODEs of this kind is to multiply both sides by an *integrating factor*, $e^{\alpha t}$:

$$\dot{h}\ e^{\alpha t} + \alpha\ h\ e^{\alpha t} = \frac{I}{A}\ e^{\alpha t} \tag{4.60}$$

We use the integrating factor because the chain rule is:

$$\frac{d(u\ v)}{dt} = \frac{du}{dt}\ v + u\ \frac{dv}{dt} \tag{4.61}$$

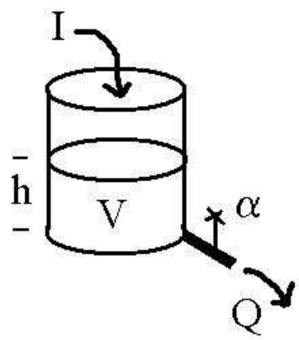


Figure 4.8: Linear reservoir with a constant input.

which, for our case, is:

$$\frac{d(h\ e^{\alpha t})}{dt} = \dot{h}\ e^{\alpha t} + h\ \left(\alpha e^{\alpha t}\right) \tag{4.62}$$

Substitution yields:

$$\frac{d(h\ e^{\alpha t})}{dt} = \frac{I}{A}\ e^{\alpha t} \tag{4.63}$$

so that:

$$d(h\ e^{\alpha t}) = \frac{I}{A}\ e^{\alpha t}\ dt \tag{4.64}$$

Integrating both sides yields:

$$h\ e^{\alpha t} = \frac{1}{\alpha}\ \frac{I}{A}\ e^{\alpha t} = \frac{I}{k}\ e^{\alpha t} + C \tag{4.65}$$

Dividing both sides by the integrating factor, and inserting the initial condition, $h = h_o$ at $t = 0$, yields:

$$h = h_o\ e^{-\alpha t} + (1 - e^{-\alpha t})\ \frac{I}{k} \tag{4.66}$$

General Routing Equation

Rather than use simple conceptual models, we can formulate a fundamental physical model that describes channel flows. Once this physical model has be constructed, we can then apply it for the routing flood flows through streams.

The first equation we use is the conservation of mass equation:

$$\Delta Q = O - I = -\frac{\Delta S}{\Delta t} \tag{4.67}$$

Where ΔQ is the change in discharge along the channel, O is the channel outflow, I is the channel inflow, S is the amount of water in storage in the channel section, and $\Delta S/\Delta t$ is the change in storage over time. This equation means that when inflows exceed outflows, the amount of water in storage must increase over time.

We can write this as a one-dimensional partial differential equation (PDE):

$$\frac{\partial Q}{\partial x} = -\ \frac{\partial A}{\partial t} \tag{4.68}$$

where the left-hand side represents the change in flow per unit length of channel, and the right hand side represents the change in channel cross-sectional area, A, per unit time.

We can approximately this equation using *finite differences*:

$$\frac{\Delta Q}{\Delta x} = -\ \frac{\Delta A}{\Delta t} \tag{4.69}$$

so that:

$$\Delta Q = -\ \frac{\Delta S}{\Delta t} \tag{4.70}$$

where $\Delta Q = O - I$ and $\Delta S = \Delta A\ \Delta x$. Note that this is identical to the original equation.

Using the original equation, we can define inflows, outflows, and storages as the averages between two times:

$$\bar{O} = \frac{O(t + \Delta t) + O(t)}{2} \tag{4.71}$$

$$\bar{I} = \frac{I(t + \Delta t) + I(t)}{2} \tag{4.72}$$

$$\Delta S = S(t) - S(t + \Delta t) \tag{4.73}$$

Substitution yields:

$$\frac{O(t) + O(t + \Delta t)}{2} - \frac{I(t) + I(t + \Delta t)}{2} = \frac{S(t) - S(t + \Delta t)}{\Delta t} \tag{4.74}$$

If we are monitoring streamflows, we have the current observations of $I(t)$, $O(t)$, and $S(t)$. We can also obtain predictions for the upstream inflows, $I(t + \Delta t)$. Solving for the future downstream outflow yields:

$$O(t+\Delta t) = I(t+\Delta t) + I(t) - O(t) - \frac{2}{\Delta t}\ [S(t + \Delta t) - S(t)] \tag{4.75}$$

But we are still have one unknown on the right-hand side of the equation, $S(t+\Delta t)$. To eliminate this variable, we require a relationship between S and O.

Continuity equation. One thing we know that might help us is that we can't create or destroy matter. This conservation of mass gives us a *mass balance* equation:

$$I - O = \frac{\Delta S}{\Delta t} \tag{4.76}$$

where I and O are the inflows and outflows, respectively, and $\Delta S/\Delta t$ is the change in storage over time.

This can be written in differential form using:

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0 \tag{4.77}$$

This means that a decrease in flow observed with distance results in an increase in the cross-sectional area with time.

One can also write this as:

$$\frac{\partial(Av)}{\partial x} + \frac{\partial A}{\partial t} = 0 \tag{4.78}$$

$$v\,\frac{\partial A}{\partial x} + A\,\frac{\partial v}{\partial x} + \frac{\partial A}{\partial t} = 0 \tag{4.79}$$

For a channel of constant width, i.e., $A = wh$, this is:

$$v\,\frac{\partial h}{\partial x} + h\,\frac{\partial v}{\partial x} + \frac{\partial h}{\partial t} = 0 \tag{4.80}$$

so that the water level is used as the primary variable (along with v).

Alternatively, we note that $c = dQ/dA$, so that:

$$c\,\frac{\partial A}{\partial x} + \frac{\partial A}{\partial t} = 0 \tag{4.81}$$

which is just:

$$c\,\frac{\partial h}{\partial x} + \frac{\partial h}{\partial t} = 0 \tag{4.82}$$

for a channel of constant width and celerity.

Nonlinear Outflow-Storage Equation. One approach is to use a nonlinear relationship between S and O, which would be consistent with the weir equation, $O = C\ S^n$, where C is the weir coefficient, and $n = 3/2$ for a rectangular weir. Solving for S yields the nonlinear storage-outflow relationship, $S = (O/C)^{2/3}$.

Substituting the storage-outflow relationship and solving for the future outflow yields:

$$O(t+\Delta t) = I(t+\Delta t) + I(t) - O(t) - \frac{2}{\Delta t}\left[\left(\frac{O(t+\Delta t)}{C}\right)^m - S(t)\right] \tag{4.83}$$

This must be solved iteratively, using an initial estimate of $O(t+\Delta t)$ on the right-hand side to obtain a revised estimate on the left-hand side, which is then used on the right-hand side until the value converges.

Muskingum Method. An alternative, early approach for estimating the storage function was an empirical method that splits the storage into two parts, a steady, *prism* storage, and a transient, *wedge* storage:

$$S = S_{prism} + S_{wedge} \tag{4.84}$$

where

$$S_{prism} = \tau\ O \qquad S_{wedge} = x\ \tau\ (I - O) \tag{4.85}$$

where τ is the flood wave travel time and x is a weighting factor, $0.1 < x < 0.5$, that indicates the amount of flood attenuation. Substitution yields:

$$S = \tau\ O + x\ \tau\ (I - O) = \tau\ [x\ I + (1-x)\ O] \tag{4.86}$$

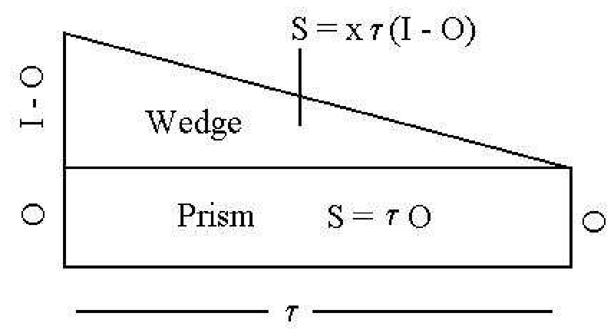


Figure 4.9: Sketch diagram for Muskingum storage.

Substituting this into the finite difference equation yields:

$$O(t+\Delta t) = C_1\ I(t+\Delta t) + C_2\ I(t) + C_3\ O(t) \tag{4.87}$$

where

$$C_1 = \frac{\Delta t - 2\tau x}{\Delta t + 2\tau(1-x)} \tag{4.88}$$

$$C_2 = \frac{\Delta t + 2\tau x}{\Delta t + 2\tau(1-x)} \tag{4.89}$$

$$C_3 = \frac{-\Delta t + 2\tau(1-x)}{\Delta t + 2\tau(1-x)} \tag{4.90}$$

Note that $C_1 + C_2 + C_3 = 1$. The value of x can be determined using regression.

Muskingum-Cunge Method. We can directly obtain the value of x used in the Muskingum equation by 1) adding lateral inflows, q_L, and 2) substituting the kinematic velocity, $c = dQ/dA$, into the original PDE, yielding:

$$\frac{\partial Q}{\partial x} + \frac{1}{c}\,\frac{\partial Q}{\partial t} + q_L = 0 \tag{4.91}$$

where $q_L = Q_L/\Delta x$ is the lateral inflow rate, Q_L, per unit length of stream channel, $\Delta x = x_b - x_a$. This is the same as:

$$c\,\frac{\partial Q}{\partial x} + \frac{\partial Q}{\partial t} + c\ q_L = 0 \tag{4.92}$$

This yields a physically-based routing equation that can be solved to yield:

$$O(t+\Delta t) = C_1\ I(t+\Delta t) + C_2\ I(t) + C_3\ O(t) + C_4 \tag{4.93}$$

where C_1, C_2, and C_3 are identical with the Muskingum Method, but now:

$$C_4 = \frac{q_L\ \tau\ \Delta t}{\Delta t + 2\tau(1-x)} \tag{4.94}$$

and

$$x = 0.5\ \left(1 - \frac{q_L}{c\ \Delta h}\right) \tag{4.95}$$

where $\Delta h = h_b - h_a$ is the head drop between the upstream and downstream locations.

Muskingham Variants. A variant of the method, called the *Muskingum Crest Segment Routing* method, uses only past inflows to the system by replacing $O(t)$ with previous predictions, i.e.:

$$O(t) = C_1\ I(t) + C_2\ I(t-\Delta t) + C_3\ O(t-\Delta t) \quad (4.96)$$

and

$$O(t-1) = C_1\ I(t-1) + C_2\ I(t-2\,\Delta t) + C_3\ O(t-2\,\Delta t) \quad (4.97)$$

and so on. Substituting these into the original Muskingum equation yields:

$$O(t+\Delta t) = \sum_{i=o}^{n} K_i\ I(t-i) \quad (4.98)$$

$$= K_o\ I(t) + K_1\ I(t-1) + K_2\ I(t-2) + ... \quad (4.99)$$

where

$$K_o = C_1 \quad (4.100)$$

$$K_1 = C_1\, C_3 + C_2 \quad (4.101)$$

$$K_2 = C_3\ K_1 \quad (4.102)$$

$$K_n = C_{n+1}\ K_{n-1} \quad (4.103)$$

A problem with this method is that it ignores accumulating errors that might have been corrected by incorporating previous outflows.

Yet another method is the *SCS Convex* method that is a simplification where $C_1 = 0$, $C_3 = 1 - C_2$, so that:

$$O(t) = I_1 + C_2\ [I(t-1) - O(t-1)] \quad (4.104)$$

The assumption that $C_1 = 0$ implies that $\Delta t = 2\tau x$. This is clearly the case when $x = 0.5$ and $\Delta t = \tau$. One might select a time step which satisfies this condition.

Problems

1. What does a kinematic ratio of 0.7 tell you about the flood potential downstream?
2. Go to the Georgia U.S. Geological Survey website, wwwga.usgs.gov
   (a) Download discharge data for the Middle Oconee River at Arcade.
   (b) Download discharge data for the Middle Oconee River at Athens.
   (c) Import the two datasets into a spreadsheet where Column A is the date/time, Column B is the Arcade discharge, and Column C is the Athens discharge.
   (d) Plot the two hydrographs for a stormwater period on the same graph.
   (e) Calculate the Muskingum coefficients using regression for the storm.
3. Fill your bathtub with water
   (a) You can take a bath before you start the experiment so you don't waste the water
   (b) When you are done, get out, and measure the depth of water in the bathtub.
   (c) Open the drain and record the time.
   (d) Every 30 seconds or so, record the time and the depth of water.
   (e) Enter the data into a spreadsheet. Put the elapsed time in Column A and the depth in Column B.
   (f) Plot the depth as a function of time.
   (g) Fit a power trend line to data.
4. Use data of your choice
   (a) Estimate the lagged covariance function between the independent in dependent variables.
   (b) Estimate the lag and memory of the covariance function.
   (c) Estimate the Unit Response Function.
   (d) How well does the fitted model fit the observed data?